Monetary Theory, Institutions and Policy

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Contents

Chapter 1

Money and its Functions

1.1 Preamble

These are lecture notes were taken during the winter semester (2025-2026) at the Economics Department of Roma Tre University, of the *Monetary Theory, Institutions and Policy* course, adapted from the slides (and class notes) of professor Sergio Levrero.

1.2 First Definitions

We begin by defining money through it's historic chain of appearances, as seen in Table 1.2. Nevertheless, economists have followed a *functional approach* for the definition of money: "money is what money does".

The functions of money are the following: (*i*) unit of account: it is the way in which debts and price lists are expressed; (*ii*) medium of exchange; and (*iii*) reserve of value. Greater emphasis is placed on one or another of this functions depending of the theory we choose to study the subject. **Neoclassical theory puts emphasis on money as means of exchange. Keynesian theory puts emphasis on money as a fund of value.**

	Money-of-Account	
	↓	
Money-Proper	and	Acknowledgment of Debts
↓		↓
State-Money → → →	↓	← ← ← Bank-Money
↓	↓	
↓	Representative-Money	→ → → Fiat Money
↓	↓	
Comomodity-Money →	Managed-Money	

Table 1.1: A Historic Diagram of Money

1.3 The Origins of Money and Different Emphasis on it's functions

The prevailing interpretation on the origins of money **focuses on its function as a means of exchange**: this position argues that its creation is related to the necessity of reducing transactions costs, compared to those of a barter economy: where exchange only occurs in the light of a double coincidence of needs and goods. This was implicit in Smith (2019), where he highlighted the advantages of monetary economy. These advantages persist as long as there is a relative stability of the purchasing power of the monetary standard.

Keynesian and Post-Keynesian authors tend to **focus on money as a unit of account** to explains the origins of money. Money antecedes writing, and the firs examples of writing precisely appear in the context of taking debts and credits into record. Primitive societies did not trade goods, rather they used them as gifts and debts (marriages, killings, punishments), even though this not occurred in an standardized systematic fashion.

Forms of social stratification emerged in the historic evolution of societies, given place to the development of tributes towards certain branches of society: this tributes where paid in an specific unit of account defined by a "sovereign". The first examples of minting of coin-money did not appear necessarily in the context of trade (see the *tallies* of medieval

Europe), but rather as payment tokens and evidence of debt. Money as a durable, and portable, debt, introduces us to the notion of debt towards the state, and the consensus of what is the legal title that releases an entity from a contract of debt: the state then claims the right to define what should answer as money to the current money-of-account that arose from the evolution of society. **This interpretation on the origins of money concludes that *money is debt, but not all debts are money*:** "it is what the state defines as unit-of-account and to which it attributes the power to free from debt and pay taxes".

There is an efficiency aspect attached to this specific function of money (as a unit of account). Taking a commodity as a numeraire of prices, the number of relative prices that we need to know is smaller than in the absence of this numeraire. Without it in exchange, where there exists n goods, the number of relative prices (pair by pair) that we will have to know are:

$$\binom{n}{k} = \binom{n}{2} = \frac{n(n-1)}{2}$$

Imagine that we have five goods: A, B, C, D and E. If we want to engage in trade, we would have to know the relative price of A/B, A/C, A/D, A/E etc. Using our formula we would have to know 10 relative prices. We quote every price in terms of A, then we only need $n-1$ prices: B/A, C/A, D/A and E/A. The choice of numeraire had effects on the *real value* of money in the past, we find two approaches (described by Pigou): (i) subjective: based on the quantity theory of money, the real value of goods is given by $p_{real} = p_{nominal}\frac{1}{p_g}$ where $\frac{1}{p_g}$ is the reciprocal of the general price level (as function of the quantity of money in circulation); (ii) objective: in terms of production costs, so that the value of goods depends on the methods of production (the technical conditions of the various commodities in existence and the numeraire) an the ruling wage rate.

This implies that the price level will change depending on the numeraire we choose. Different monetary standards have effects that affect the debt-credit relationships: inflation is favorable for debtors and deflation is favorable to creditors.

Focusing on money as a reserve of value introduces the notion of liquidity: financial assets have different "degrees" of immediate spendability and rapid convertibility.

Money also functions as a way to "transfer" wealth over time, through the acquisition of financial and real assets. The notion of liquidity further complicates the creation of a clear cut definition of which aggregates distinguish money from other financial assets and real assets.

This is mostly accomplished by emphasizing money's capacity to transform immediately into real resources without costs for it's transfer and purchase. **The cost of money is, from this focus (as a reserve of value), defined by its opportunity cost: the cost of *giving up liquidity.*** It's possible to define different monetary aggregates by including activities with lower degrees of liquidity, and different degrees of risk: making them more or less substitutable with each other and with money.

We can measure this *substitutability* by the cross elasticity of demand for an asset with respect to the change in the interest rate of a competing asset. Let A be a fairly liquid financial asset, which we can compare with a slightly less liquid financial asset called B, then we have that:

$$\epsilon = \frac{\Delta A/A}{\Delta r_B/r_B}$$

The greater the reactivity of A to changes in r_B, in absolute value, the more these to assets are substitutable with each other. If the (negative) elasticity is low, the two activities are not very substitutable with each other. Which would mean that changes in r_B don't have a negative effect in the demand of A. The latter scenario would imply that money could be defined as a *unicum*: currency plus overnight deposits. This introduces us to the definitions of a various set of *monetary aggregates*, each one larger, more complex and less liquid than the aggregate the precedes it. We have: (*i*) $M0$: the monetary base (currency); (*ii*) $M1$: $MO+$ overnight deposits; (*iii*) $M2$: $M1+$ saving deposits; (*iv*) $M3$: $M2+$ short run activities; (*v*) $M4-M5$: $M3+$ activities and savings of long term maturity.

1.4 Monetary Base, Monetary Policy Instruments and the Institutional Setting of the European Monetary Union

1.4.1 Central Banks: historic functions

The development of credit and banking money created the necessity of regulation of the monetary circulation by the state, so, central banks started to arose in the late XVII century. **The functions of central banks are:** (*i*) funding the state (minimizing the cost of paying the public debt); (*ii*) regulating the volume of money and credit (paper money is easily created and adapts quickly to the needs of the economy); and (*iii*) acting as a *lender of last resort* (accepts deposits, grants advances to the banking sector). So, summarizing: central banks (historically) finance the state, defend the purchasing power of the currency, regulate the bank system, and, implicitly, regulate the exchange rate.

1.4.2 Monetary Base: Channels of Creation

The monetary base (BM) consists of legal currency and highly liquid assets (with none costs of conversion into legal currency). Simplifying:

$$BM = C_u + R_e$$

Where C_u and R_e are the currency in circulation and the reserves (deposits) of private banks in the CB. **The demand of BM is defined by:**

$$BM^d = BM_p + BM_b$$

Where BM_p and BM_b are the quantities of BM held by the public and the banking system to affront their liquidity necessities. This quantities may be affected by the amount of bank deposits, interest rates levels on deposits and other money market interest rates. **The supply (channels of creation) of BM are:**

$$BM^s = BM_e + BM_s + BM_f$$

BM_e (where the subindex e implies *external*) refers to the **balance of payments channel**; BM_s refers to the **treasury channel** (via financing of the public sector, hence the subindex s of *state*); and BM_f which refers to the **financing of the banking sector channel**. In the BM_s channel BM is created to finance public deficits $G - T = \Delta BM + \Delta B$ where B is the stock of public bonds. This channel is *closed* in the institutional setting of the European Union (EU) (article 104 of the Treaty of European Monetary Union, and in Italy since 1981).

Quantitative easing programs that came after the 2008 crisis led to considerable purchases of public securities by the EUCB (justified by the threat of deflation and the compliance of the *rule of capital key*). Limiting this channel elevates the cost of servicing the public debt, which is understood as an strategy to lower inflation (even at the cost of employment and income levels). The treaties that established the eurozone and the EUCB specified, along it's independence, that the main objective of the institution should be price stability.

The BM_e channel is explained by changes that occur in the balance of payments (and hence in the foreign currency reserves of a given country). **The BM_f is the main channel determining the BM.** The various operations carried by the EUCB can be summarized analyzing the terms in which monetary policy decisions are made, see Figure 1.1.

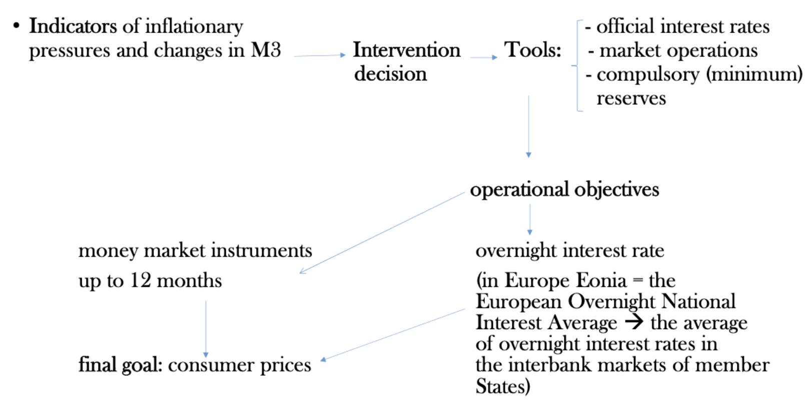

Figure 1.1: The decision (terms) structure of the European Union Central Bank.

Minimum bank reserves requirements determine the structural need for liquidity on the banking system. **Interest rates, however, are controlled through open market operations, they can be:** (i) temporary operations; or (ii) definitive operations. Temporary operations include: spot sale or purchase with forward resale, guaranteed loans, and, currency swaps. Definitive operations include *quantitative easing* activities and repo operations. Definitive operations structurally modify the liquidity conditions of the banking system. **More on temporary operations at the initiative of the CB:** (*i*) main refinancing operations (MRO): liquidity auctions, with fixed or variable, rates on a weekly frequency (with a maturity of two weeks); (*ii*) long term refinancing operations: carried out monthly (with a maturity of three months); and (*iii*) fine tuning: temporal in nature, they deal with unexpected liquidity fluctuations.

We can also find transactions at the initiative of the counter-parties (private banks): (*i*) marginal refinancing operations (discount counter, access conditional to higher rates); (*ii*) overnight deposits at the CB. Main refinancing operations are carried out through auctions. Fixed rate auctions lead to overbidding, because the liquidity to be distributed between the bidders is divided proportionally (the bank sets a rate that gets modified with supply and demand interactions). In variable rate auctions the bank establishes a minimum interest rate, banks then propose up to ten proposal to buy liquidity at different rates.

The average of this minimum rates (in variable auctions) together with the main refinancing rate of fixed rate auctions, influences the overnight rate of the the interbank market. We can now define the *interest rate corridor*, in which interbank market rates lie. We identify three main rates establishing this *corridor*: (*i*) i_d (overnight deposits rate); (*ii*) i_p (the MRO weekly auctions related rate, policy rate); and (*iii*) i_m (the marginal lending facility rate, the discount counter). Notice that

$$i_d \leq i_p \leq i_m$$

1.5 The Structure of Interest rates

CB's influence short-term (ST) interest rates, varying the rates of the marginal lending facility rate (i_m), the MRO rates (i_p) and the overnight deposits rates (i_m). Usually, **changes in ST interest rates induce changes in the same direction in long-term (LT) interest rates.** The latter ones are relevant in the formation of investment decisions and durable consumption. It is on the interest of the CB to control the relationship between ST and LT interest rates. This relationship tends to hold both for nominal and real interest rates.[1]

However, interest rates on securities of different durations are determined on the basis of supply and demand of other securities, whose demand in turn is influenced by contextual factors such as wealth, expectations, risk premia's, the degree of liquidity of the security, etc. The interest rates of different assets with different durations will tend to move in the same direction: the variation of the interest rate of an asset will have effects on the demand of other assets. **Once interest rates have been defined, it is possible to determine a yield curve: the yield curve represents the structure of interest rates.**

The simplest way of determining this curve is to express LT interest rates as a function of ST rates, plus other exogenous variables. This is a reduced form of expressing the relationship between assets of different maturities without the need to specify the supply and demand conditions of all the markets for these various securities. There are three theories, or paths, of achieving this goal.

$$(i)\ (1+R)^n = \prod_{i=1}^{n}(1+r_i) \Rightarrow R = \frac{\sum_{i=1}^{n} r_i}{n} \text{ (expectations theory)}$$

$$(ii)\ R_{nt} = \sum_{i=1}^{n} \alpha_i r_{t-i} + \sum_{i=1}^{n} \beta_i z_i \text{ (preferred habitat theory)}$$

$$(iii)\ (1+R)^n = \prod_{i=1}^{n}(1+r_i) \times k_n \Rightarrow R = \frac{\sum_{i=1}^{n} r_i}{n} + k_n \text{ (liquidity premium theory)}$$

Theories (*i*) and (*iii*) are pretty similar. Both express the LT interest rate R as an average of the expected uniperiodal interest rates, theory (iii) adds a liquidity premium k_n: borrowers prefer the LT, while lenders prefer the ST. Theory (i) cannot explain increasing yield curves. **Theory (*ii*) expresses the LT interest rate of a duration of n periods, at time t, as a weighted average of n lagged short term interest rates** (extrapolative expectations), plus a weighted average of the risk premiums for n periods.

According to theory (i), $(1+R)^n = \prod_{i=1}^{n}(1+r_i) \Rightarrow R = \frac{\sum_{i=1}^{n} r_i}{n}$. This past equalities can be read as a *non-arbitrage condition*: given the expectations of future uniperiodal interest rates, then, an investor must be indifferent (in expected return terms) between holding a

[1]Remember the so called *Fisher Effect*: $r_t \approx i_t - \pi^e_{t-1}$.

LT bond and rolling over a sequence of ST bonds. The expected gross return from buying and holding an n-period bond equals the expected gross return from rolling over one-period bonds for n periods. From this result, it is also evident that forward rates (future expected ST rates) are implicit in the observed LT rates. The ratio $\frac{(1+R_n)^n}{(1+R_{n-1})^{n-1}} = (1 + r_n) \Rightarrow r_n \approx R_{n-1} + n(R_n - R_{n-1})$.

An inverted yield curve is an unusual situation in which ST rates display higher yields relative to LT rates. Possible causes behind an inverted yield curve: if R is, in any sense, a function of future expected ST interest rates, a fall in the latter's would imply lower LT interest rates (the CB could be lowering ST rates to fight a recession, or deflation).

The CB has then the ability, by modifying ST interest rates and influencing market expectations in relation to their future value, alter the trajectory of LT interest rates (hence, the structure of the interest rates). Both for keynesians and neoclassicals this is true in the short run. In the long run neoclassicals think that monetary policy adapts to *natural rates*, associated to the marginal productivity of capital. For Keynes there is, on the one hand, no reason whatsoever why in the long run LT interest rates could not be too high for full employment, and the other hand, he believes that there is *monetary nature* of the interest rate. Also, for Keynes, the marginal productivity of capital adjusts in relation to the money market interest, and not the other way around.

1.6 Unconventional Monetary Policies

After 2008, with the decoupling between short-term (ST) and long-term (LT) interest rates and the growing difficulty of keeping borrowing costs low, central banks developed a new set of monetary tools to influence the entire structure of interest rates. They intervened through: (*i*) forward guidance, by revealing their expected path of future policy rates; (*ii*) full-allotment refinancing operations, supplying unlimited liquidity at a fixed rate; and (*iii*) the expansion of central bank balance sheets via the purchase of public and private assets—this last instrument known as Quantitative Easing (QE).

Quantitative Easing is considered "unconventional" because it departs from

the traditional mechanism of monetary policy, which operates mainly through changes in short-term interest rates. Instead, QE aims to lower long-term interest rates by artificially raising the price of long-term securities and bonds, thereby reducing their yields. By injecting liquidity directly into the financial system, QE seeks to stimulate borrowing and investment and counter deflationary pressures, particularly in situations where the central bank can no longer reduce its short-term policy rate—such as when the economy risks falling into a liquidity trap.

1.6.1 The Corridor and Floor Systems

In the corridor system we have that $i_d \leq i_p \leq i_m$, this rates correspond to the overnight deposits rate, the policy rate (associated to the MRO), and the marginal lending facility rate. We also have that the overnight market rate (the rate at which banks lend to each other) lies in $i_d \leq i_o \leq i_m$. Let R_D be the amount of reserves that the bank system demands for precautionary reasons and payment settlements, and let R_S be the available amount of reserves. This implies that:

$$(i) \text{ If } R_S = R_D \Rightarrow i_o = i_p$$

$$(ii) \text{ If } R_S < R_D \Rightarrow i_d \leq i_p \leq i_m$$

$$(iii) \text{ If } R_S > R_D \Rightarrow i_o = i_d$$

In (i) when the CB drains the excess reserves to zero (the supply and the demand for reserves are equal), i_o equals the target/policy rate associated to MRO's. The equilibrium condition for normal liquidity is $i_o = i_p$. In (ii) there is an scarcity of reserves, as competition for liquidity in the interbank market rises, i_o rises: if competition provoke that $i_o > i_p$ then banks turn to MRO's until $i_o = i_p$. Finally, as in (iii), if $R_S > R_D$, then banks have no reason to borrow reserves for each other in the interbank market (the demand for liquidity in the market collapses), nor do they have the necessity of engaging in MRO's (this reduces the cost of operations of the CB).

Given that no bank demands liquidity, no bank would lend to another bank for less than what it can earn risk-free from the CB at i_d, this pins down i_o to i_d. In the floor system, one the contrary, we find that $i_d = i_p = i_o$, changes in R_S do not alter interest rates:

banks are indifferent to the amount of available reserves. If by any chance it would occur that $i_o > i_d = i_p$ then the banks would offer the exceeding reserves, dropping i_o back to equilibrium.

Chapter 2

The Demand for Money

2.1 The Quantity Theory of Money

The Quantity Theory of Money (QTM) was first formulates in the XVI century, and by David Hume in the XVIII. It can be represented by the following system of equations:

$$M_s = M_d \tag{2.1}$$

$$M_s = M_s^* \tag{2.2}$$

$$M_d = kPY \tag{2.3}$$

$$Y = Y^* \tag{2.4}$$

$$k = k^* \tag{2.5}$$

Where M_s is the supply of money (exogenously given by the monetary authorities); M_d is the demand for money; k is the reciprocal of the velocity of circulation of money (v); and Y is the real income. k and Y are considered given and independent of each other and M_s. Equation (2.4) is justified via the neoclassical assumption of full utilization of productive resources (full employment) under the condition that supply and demand freely operate in the economy (in earlier versions of the theory authors used T, instead of Y, to refer to the total amount of transactions in an economy). Equation (2.5), was, in the first versions of the theory assuming that payment schemes and conventions where constant. Latter this constancy was justified via the desire of the public to hold liquid balances.

This implies that $PY = M_s^* V$. **This is the *exchanges equation.* It is an identity that says the the monetary expenditure (nominal income) is equal to the quantity of money** (M_s) **multiplied by the velocity of circulation of money** (V). The system is closed, solving for P we have that:

$$P = \frac{M_s^*}{k^* Y^*} \tag{2.6}$$

The last expression means that **if k, Y and M_s are constants, the general level of prices tends to be a proportion of M_s.** If $M_S > M_d$, then prices will increase, which in turn will increase the demand for money in nominal terms until $M_S = M_d$.

2.1.1 Friedman's Rehabilitation of the QTM

For Friedman, money is demanded for it's utility as a mean of exchange and the *security* and *pride* that derives from it's possession. Money is a *unicum* that produces an specific utility. **The main thesis may be summarized as followed: the demand for money is a stable function of real income.**

Given agents preferences u, the demand for money is affected by: (i) overall private wealth W, where $W = Y_p/r$ (the actual value of permanent income Y_p); (ii) human capital; (iii) the proportion of non-human capital in overall wealth w; (iv) r_b and r_e (returns on bonds and real capital, respectively); and (iv) their appreciation, or depreciation in nominal terms $\frac{1}{P}\pi$ where P is the general level of prices (such that $\frac{1}{P}$ is the price of money) and the rate of inflation $\pi = \frac{dP}{dt}$. Under the assumption that the dispersion of individual money demands around the average is constant over **we have an *implicit function* for the aggregate demand of money:**

$$M_d = f(P, r_b, r_e, \frac{1}{P}\pi, Y_p, w, u) \tag{2.7}$$

Friedman assumes the *absence of money illusion*, which implies that $\frac{M_d}{P}$ remains constant even when P changes. Hence 2.7 is a homogeneous linear equation of degree one in P. Let $\lambda = \frac{1}{P} \Rightarrow$

$$\frac{M_d}{P} = \lambda M_d = f(\lambda P, r_b, r_e, \frac{1}{P}\pi, \lambda Y_p, w, u) = f(r_b, r_e, \frac{1}{P}\pi, Y_{rp}, w, u) \tag{2.8}$$

This last expression is the demand for money in real terms as a function of real permanent income Y_{rp}, primarily: Friedman assumes that M_d is not very elastic with respect to the other variables (particularly in regards to r_b and r_e). The justification: (i) money is a unicum with an specific utility, not very replaceable with other financial assets; and (ii) empirical studies showing a relative stability of v. Letting $\lambda = \frac{1}{Y_p} \Rightarrow$

$$\frac{M_d}{Y_p} = \lambda M_d = f(\lambda P, r_b, r_e, \frac{1}{P}\pi, \lambda Y_p, w, u) = f(\frac{P}{Y_p}r_b, r_e, \frac{1}{P}\pi, w, u)$$

$$\Rightarrow Y_p = \frac{M_d}{f(\frac{P}{Y_p}r_b, r_e, \frac{1}{P}\pi, w, u)} \text{ let } v = \frac{1}{f(\frac{P}{Y_p}r_b, r_e, \frac{1}{P}\pi, w, u)}$$

$$\Rightarrow M_d = kY_p = kPY_{rp} \tag{2.9}$$

$$\Rightarrow \frac{M_s}{P} = \frac{M_d}{P} = k\frac{Y_p}{P} = kY_{rp} \tag{2.10}$$

This restatement of the QTM in terms of choices and behaviors of agents, depends on: (i) $V = \frac{1}{k}$ is stable except for particular circumstances; and (ii) the permanent income in real terms tends to the level of full employment. Equation (2.10) expresses the equilibrium in the money market in real terms, **the adjustment between M_s and M_d takes place through changes in P (and not in the interest rate, as in Keynes).** If $\frac{M_s}{P} > \frac{M_d}{P}$, remembering that k and Y_{rp} don't fluctuate, then P increases until $\frac{M_s}{P} = \frac{M_d}{P}$.

The endogeneity of P, plus the result of M_d being a stable proportion of Y_{rp} Friedman concludes that: (i) there is mechanism of monetary policy that doesn't pass through changes in the interest rate; (ii) that the money multiplier of permanent income equals the velocity of money ($\frac{dY_p}{dM_s} = V_p$); (iii) that this multiplier is stable; and (iv) the oscillations of v around v_p have a role in determining the oscillations of Y around Y_p (however, in the long run all this tends to cancel out).

2.1.2 Controversies with Keynesian Theory

Keynesians elevated two main criticism against the QTM that Friedman exposed: (i) one regarding the sensitivity of M_d with respect to Y and r; (ii) one regarding the assumption of v being stable over time. For example, Kaldor argued that the apparent stability of v might not be explained by a relative stability on M_d, but rather an instability of M_s (so that money supply adapts to M_d, i,e., M_s is endogenously determined).

The QTM also fails to specify: which monetary aggregates to consider; which rates to consider (ST, LT); what is the scale variable (financial wealth, level of income); and how to consider returns on real assets. However, the one aspect that Keynesians found more controversial with this theory, has to do with the paradox of explaining how could monetary policy be effective in a world where M_d is inelastic with respect to r.

2.2 The Demand of Money as a Reserve of Value

2.2.1 Securities and the Market Rate of Interest

Money transfers wealth over time, it's nominal value always stays constant, however, money does not offer any return (even though it has a high degree of transferability). The choice of instrument in order to transfer wealth over time is influenced by: their transferability; the cost of transfer; the certainty of their nominal and real value; and the rate of return of alternative instruments.

Securities provide a return, but do not ensure any constancy related to their nominal value. **Bonds, both with a constant annuity R or or with a constant annuity with a maturity of n years hold an *inverse relationship* between their value (v) and the market rate of interest (r).** Observe that:

$$v_c = \frac{cP}{r} = \frac{R}{r} \tag{2.11}$$

$$v_n = \frac{cP}{r}\left[1 - \frac{1}{(1+r)^n}\right] + \frac{P}{(1+r)^n} \tag{2.12}$$

Where c is the rate set at the emission of the security and P is the nominal value, or face value, of the security. Equation (2.11) represents the value of and irredeemable bond with a constant annuity R, while equation (2.12) represents the value of a bond with a constant annuity with a maturity of n years. **The value of both bonds changes inversely to the direction of r.**

2.2.2 Keynes's Liquidity Preference

Keynes distinguishes three motives to hold money: (*i*) the transactional motive; (*ii*) the precautionary motive; (*iii*) and the speculative motive. The first two motives are still explained in a similar fashion as the one exposed by Lavington and the so called "Cambridge School". The nominal demand for money for transactional motives is given by Keynes by $L_1 = kY$ where Y is the nominal income. **For Keynes money represented a link between present and future value (it guarantees no capital losses), the price of money is then the observed rate of interest: that is, the *opportunity cost of holding money* (or liquidity).** For agents, this implies that:

$$r \leq r_e \iff p_e \leq p \text{ (demand money)}$$

$$r > r_e \iff p_e > p \text{ (buy securities)}$$

Where r is the current market interest rate, r_e the expected market interest rate, p is the face value of a given asset and p_e is the expected value of that asset. The relations described in Subsection 2.2.1 indicate that an expectation in the increase of the interest rate will entail for the agent the symmetrical expectation of a decrease in the price of securities: hence creating an incentive to hold his wealth in money to avoid capital losses. And vice-versa if the interest is expected to fall.

We can further illustrate this mechanism with a simple portfolio selection model, with two key assumptions: (*i*) agents have a definitive expectation of r; and (*ii*) there exists a dispersion of the expectations among agents. Agents are individuals or financial institutions seeking capital gains due the rise or fall in the prices of securities due to changes in the interest rate. Notice how, even with heterogeneous r_e's, as the interest rate goes down the number of agents expecting an increase in the interest rate in the future increases: **hence there exists an inverse relationship between the interest rate** (r) **and the demand for money for speculative motives** (L_2). In this framework agents will choose to hold *all* their wealth in either money or securities (*corner solution*).

Let A_1 and A_2 the percentages that express the relative weight of money and securities in the agents portfolio such that:

$$0 \leq A_1 \leq 1$$

$$0 \leq A_2 \leq 1$$

$$A_1 + A_2 = 1$$

Money doesn't pay any interests, and bonds pay a rate r. For simplicity we assume that all bonds are perpetuities such that $p = \frac{1}{r}$. With this information we can calculate the expected and current capital gains g_0, g_e:

$$g_0 = \frac{p_0 - p_{-1}}{p_{-1}} = \frac{\frac{1}{r_0} - \frac{1}{r_{-1}}}{\frac{1}{r_{-1}}} = (\frac{r_{-1}}{r_0}) - 1 \tag{2.13}$$

$$g_e = \frac{p_e - p_0}{p_0} = \frac{\frac{1}{r_e} - \frac{1}{r_0}}{\frac{1}{r_0}} = (\frac{r_0}{r_e}) - 1 \tag{2.14}$$

Where r_0 is the current market interest rate and r_e is the expected market interest rate associated to each individual agent. This of course implies that

$$r_0 = r_e \Rightarrow g_e = 0$$

$$r_0 < r_e \Rightarrow g_e < 0$$

$$r_0 > r_e \Rightarrow g_e > 0$$

Let R be the effective total effective returns of the portfolio, as a weighted average of the returns on money and bonds:

$$R = A_1 R_1 + A_2 R_2 \text{ given that } R_1 = 0$$

$$\Rightarrow R = A_2 R_2 = A_2(r_0 + g_0) \tag{2.15}$$

$$\Rightarrow R^e = A_2 R_2^e = A_2(r_0 + g_e) \tag{2.16}$$

R^e then is defined as the expected returns of the portfolio given an specific r_e (and hence also a g_e). Notice how R and R^e not only depend on r_0, but also on g_0 and g_e. **If $R_2 < R_1$ (such that $R_2 < 0$) then the agent will prefer to hold his/her wealth in money, an conversely if $R_2 > 0$ the agent will prefer to hold government bonds.** Letting $R^e = 0$, that is, letting $r_0 + g_e = 0$ we can calculate the value of r_0 that makes the agent indifferent between holding their wealth in money or in bonds:

$$r_c = \frac{r_e}{(1 + r_e)} \tag{2.17}$$

Equation (2.17) describes the *critical rate of interest*, associated to an specific r_e. The critical rate of interest is the *present value* of the expected interest rate. This entails three possible scenarios:

$$\begin{aligned} &(i) \text{ If } r_0 > r_c \Rightarrow R^e > 0 \text{ (the agent chooses bonds)} \\ &(i) \text{ If } r_0 < r_c \Rightarrow R^e < 0 \text{ (the agent chooses money)} \\ &(i) \text{ If } r_0 = r_c \Rightarrow R^e = 0 \text{ (the agent is indifferent)} \end{aligned} \quad (2.18)$$

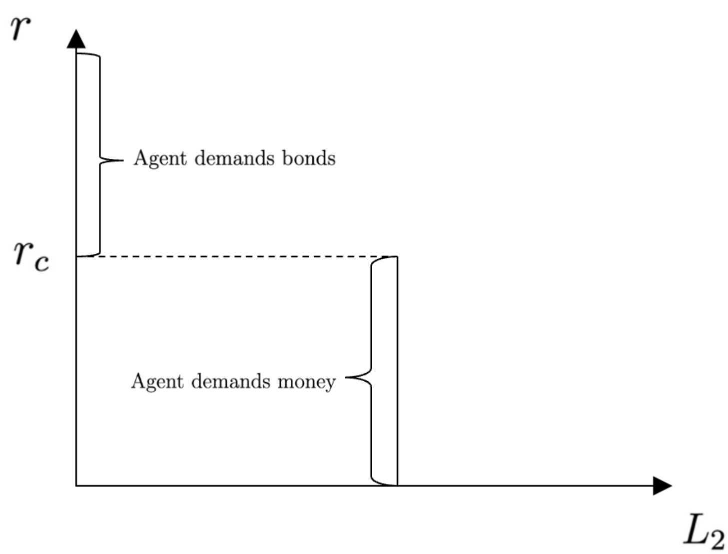

Figure 2.1: Graphical representation of the definite choice between holding money and bonds.

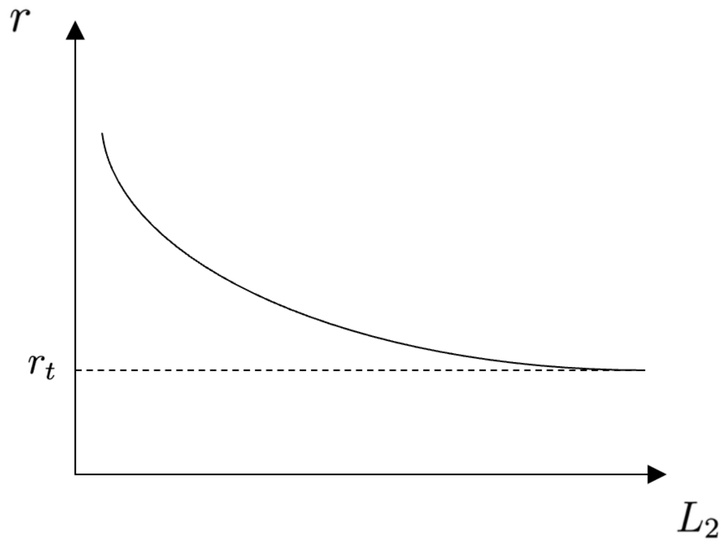

Figure 2.2: The demand curve of L_2 as an inverse relationship of r.

Graphically we can observe the speculative demand for money of an agent in Figure 2.1. **For the economy as a whole there will be an speculative demand curve for money in an inverse relationship to the interest rate.** As r_0 falls more and more agents find that $r_0 < r_c$, notice that each agent has an independent and definite r_c. There may be an r_0, that we can call r_t such that $\frac{\Delta L_2 / L_2}{\Delta r_0 / r_0} = \infty$. That is, **there is lower bound r_t that makes the elasticity of demand for money L_2 infinitely elastic with respect to the market rate r_0** (that is, we find ourselves in a *liquidity trap*). We can observe this relationship in Figure 2.2.

Given that this curve solely depends on the individual expectation regarding r, L_2 can be unstable due to changes in r_e. This contrasts with the idea of a stable demand for money in the QTE (see Section 2.1). Some criticisms have been moved against Keynes's liquidity preference theory: (i) no analysis regarding the formation of r_e; (ii) a lack of specification regarding the temporality of $r_e \neq r_0$; (iii) the assumption of r_e being independent, hence not adjusting, in relation to changes in r_0.

If r_e adjusts to changes in r then M_d will not be sufficiently independent of M_s. Thus, changes in M_s will not only imply movements along the M_d curve but a shift of the curve, Keynes needs the assumption of a constant r_e. With the recognition of an inverse relationship between M_d and r then: (i) the velocity of circulation of money cannot be considered constant (unless M_s becomes endogenous); and (ii) CB's can introduce money into the economic system by changing r. **Hence, in Keynes's liquidity preference theory M_d adjusts to M_s through changes in r, and not in the general level of prices P, as in the QTM.**

2.3 Developments in the Theory of the Demand for Money

In this section we will talk about three main developments in the Keynesian analysis of the demand for money: (i) a theory of the relationship between L_1 and i; (ii) the abandonment of the hypothesis of definitive expectations on future interest rates (hence agents might diversify their portfolio choices); and (iii) a shift from a relationship between the demand for money and nominal interest rates for a relationship of the demand for money and the *real interest rate*. We will revise two models.

2.3.1 Baumol's Model

In this model money balances can be invested and gradually dis-invested to face payment needs. **The model follows the next assumptions:** (i) income is received at regular intervals; (ii) expenditures are perfectly steady; and (iii) withdrawals of money from the bank occur at regular intervals.

The problem can be stated as follows: the agent must find the optimal withdrawal size (W, which is our *endogenous variable*), which is the one that minimizes costs (C). This, given the size of annual transactions per year (T, expenditures), withdrawal costs (b) and the interest rate r. The cost function is defined as:

$$C = b\left(\frac{T}{W}\right) + r\left(\frac{W}{2}\right) \tag{2.19}$$

In the right hand part of the equation $b\left(\frac{T}{W}\right)$ expresses the *total withdrawal costs*, while $r\left(\frac{W}{2}\right)$ expresses the opportunity cost of holding non-interest-bearing cash. Note that $k = \frac{T}{W}$

is the number of withdrawals per year and $L_1 = \frac{W}{2}$ is the average cash holdings.[1] Note that the agent needs two balance two effects in order to minimize C: the total withdrawal costs and the opportunity cost of holding liquid money. It follows that:

$$\begin{aligned} \min C &= b\left(\frac{T}{W}\right) + r\left(\frac{W}{2}\right) \\ \Rightarrow \frac{dC}{dW} &= \frac{r}{2} - b\left(\frac{T}{W^2}\right) = 0 \\ \Rightarrow W &= \sqrt{\frac{2bT}{r}} \end{aligned} \tag{2.20}$$

This implies that, as usual, the average cash holdings in the economy L_1 hold a direct relationship with the scale of the economy (T) and an inverse relationship with the interest rate (r). **Hence, an observed negative elasticity of money demand with respect to the interest rate will not only be attributed to changes in L_2 (the demand for money for speculative motives, as seen in Section 2.2.2), but also to changes in L_1.**

2.3.2 Tobin's Portfolio Selection Model

Tobin's analysis deals with the *weakness* of Keynes's liquidity preference theory regarding the fact that, in reality, there is no clear definitive choice to keep ones wealth entirely in securities or in money. **Tobin's main assumptions are:** (i) agents do not have definitive expectations about the future interest rate, instead, they express this expectations through a *Bayesian* probability distribution; and (ii) this probability is symmetrical with respect to it's central value, which is the current interest rate: $r_e \sim \mathcal{N}(r_0, \sigma_r^2)$ (see Figure 2.3).

[1] At the moment of the withdrawal cash holdings are W, this holdings are spent steadily and just before the next withdrawal (which occurs in regular intervals) the cash holdings are zero: thus on average holdings are halfway between 0 and W.

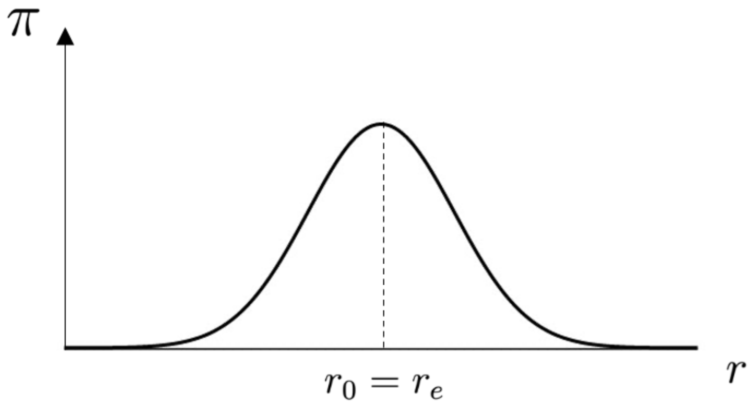

Figure 2.3: The normal-Gaussian probability distribution of the expected interest rate.

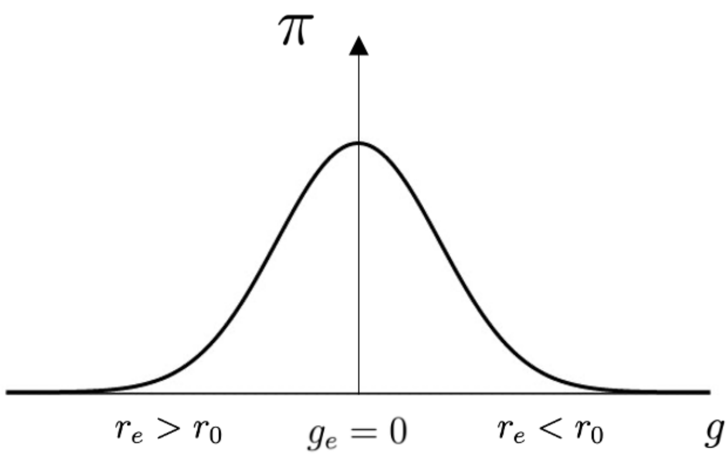

Figure 2.4: The normal-Gaussian probability distribution of the expected capital gains from the security (price variation).

The second assumption also implies that $g_e = (\frac{r_0}{r_e}) - 1 = 0$ is the central value of the probability distribution of $g_e \sim \mathcal{N}(0, \sigma_g^2)$. We can visualize this distributions in Figure 2.4. **Hence, σ_g represents the standard deviation (it is a measure of the risk of holding securities), which depends on the specific probability distribution of each agent.** Using equation (2.15) and remembering that $E(g) = 0$ we arrive a the following set of definitions:

$$
\begin{aligned}
E(R) &= E[A_2(r+g)] \\
\Rightarrow E(R) &= A_2 r \\
\sigma_R &= A_2 \sigma_g && (2.21) \\
\Rightarrow A_2 &= \left(\frac{1}{\sigma_g}\right)\sigma_R && (2.22) \\
\Rightarrow E(R) &= \left(\frac{r}{\sigma_g}\right)\sigma_R && (2.23)
\end{aligned}
$$

Equation (2.21) describes the overall portfolio risk (σ_R), which is linearly related to the proportion of securities selected by each agent (A_2), the latter defined in equation (2.22). Finally, **equation (2.23) is the definition of the *return-risk profile*, which is the objective trade-off between risk and return: there is a linear relationship between the expected return of the portfolio $E(R)$ and its risk level σ_R.** The slope of this relationship is $\left(\frac{r}{\sigma_g}\right)$, so that if $\Delta r > 0 \ \vee \ \Delta\sigma_g < 0 \Rightarrow \Delta\left(\frac{r}{\sigma_g}\right) > 0$, hence, the same $E(R)$ will be associated to a lower risk.

Now, given an expected utility function $U(E(R), \sigma_R)$, then, the investor will maximize this utility under the constraint provided by the return-risk profile. Hence, the maximization problem is:

$$\max U(E(R), \sigma_R) \text{ s.t. } E(R) = \left(\frac{r}{\sigma_g}\right)\sigma_R \tag{2.24}$$

This is a *mean-variance model*: utility increases (decreases) if the mean increases (decreases) and if variance decreases (increases). Agents indifference curves, pairs of $(E(R), \sigma_R)$ that yield the same utility, can be of three types depending on the relationship between return and risk that each agent has: (i) $\frac{dE(R)}{d\sigma_R} > 0 \ \wedge \ \frac{d^2E(R)}{d\sigma_R^2} > 0$ (*risk adverts*, increasing and convex indifference curves); (ii) $\frac{dE(R)}{d\sigma_R} > 0 \ \wedge \ \frac{d^2E(R)}{d\sigma_R^2} < 0$ (*plungers*, increasing and concave indifference curves); and (iii) $\frac{dE(R)}{d\sigma_R} < 0 \ \wedge \ \frac{d^2E(R)}{d\sigma_R^2} < 0$ (risk lovers, decreasing and convex indifference curves). The solution of the maximization problem (2.24), for a risk adverse agent, can be observed in Figure 2.5.

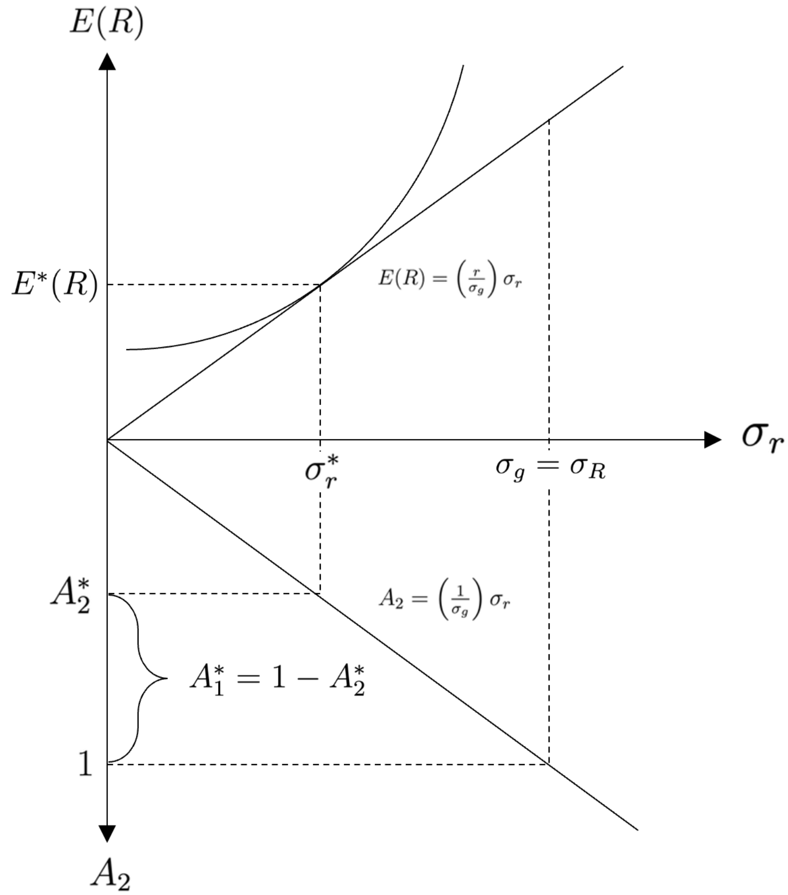

Figure 2.5: Tobin's Portfolio Selection Model.

This agent maximizes his expected utility (using the interest rate and his own subjective probability distribution of g_e) choosing the portfolio (A_1^*, A_2^*), that corresponds to the tangency point between its indifference curve and the return-risk profile: which yields an specific constrained combination of expected return $E^*(R)$ and portfolio risk σ_R^*. Also, notice that, if $\Delta r > 0 \Rightarrow \Delta A_2 > 0 \ \wedge \Delta A_1 < 0(|\Delta A_2| = |\Delta A_1|)$, as long has the *substitution effect* outweighs the *income effect*.

Chapter 3

Monetary Base, Credit and Money Supply

There are two views regarding money supply: (i) M_s is exogenous (fixed, controlled by the CB's), this is the prevailing vision in monetarist and QTM theorists; and (ii) M_s is exogenous (it ultimately depends on the needs for credit, transactions, etc), nowadays this is the *mainstream* stance, both in new-Keynesian, neoclassical, classical and post-keynesian authors.

3.1 Monetary Base and the Money Multiplier

Let's assume that the BM is only created through MRO's so that $BM^s = \overline{BM}$, the liabilities of the banking sector (for example, consumer deposits) are accepted as means of payment. **To analyze how loans and bank money are created we accept the following assumptions:** (i) the public holds a proportion γ of BM in the form of currency; (ii) the banks hold a proportion α of discretionary liquidity reserves and proportion β of mandatory reserves of BM (such that the proportion of BM in the credit circuit is $(1-\alpha-\beta)$); (iii) the previously mentioned proportions are inelastic with respect to changes in the interest rate; and (iv) the supply of credit offered by the banking system always finds a corresponding demand from households and firms.

We can calculate the amount of bank deposits (D), the monetary base held by the public

(BM_p), the monetary base held by the banks (BM_b) and the credit disbursed into the banking system (BM_c) through different circuit stages. Let $\delta = (1-\alpha-\beta-\gamma) \in (0,1)$, and let the circuit stages be $t = \{1, 2, 3, ..., n\},\ n \in \mathbb{N}$. If we assume that for $t = 1$ we have that $D = BM$, then:

$$D = BM \sum_{t=1}^{n} \delta^{t-1} = BM \frac{1-\delta^n}{1-\delta} = \frac{1}{(\alpha+\beta+\gamma)} BM \tag{3.1}$$

$$BM_p = \gamma BM \sum_{t=1}^{n} \delta^{t-1} = \gamma BM \frac{1-\delta^n}{1-\delta} = \frac{\gamma}{(\alpha+\beta+\gamma)} BM = \gamma D \tag{3.2}$$

$$BM_b = (\alpha+\beta) BM \sum_{t=1}^{n} \delta^{t-1} = (\alpha+\beta) BM \frac{1-\delta^n}{1-\delta} = \frac{(\alpha+\beta)}{(\alpha+\beta+\gamma)} BM = (\alpha+\beta) D \tag{3.3}$$

$$\begin{aligned} BM_c &= (1-\alpha-\beta) BM \sum_{t=1}^{n} \delta^{t-1} = (1-\alpha-\beta) BM \frac{1-\delta^n}{1-\delta} \\ &= \frac{(1-\alpha-\beta)}{(\alpha+\beta+\gamma)} BM = (1-\alpha-\beta) D \end{aligned} \tag{3.4}$$

This relationships hold following the definitions of the geometric series in a closed-form expression, being $S_n = ar^n = a\frac{1-r^n}{1-r}$. Note that $BM_c = D - BM_b$ and note that $BM = BM_p + BM_b = (\alpha+\beta+\gamma)D$. We can now introduce the money supply multiplier. Assuming that the relevant monetary aggregate is $M1 = C + D$, where $C = BM_p$ (the currency in circulation), we can then determine (starting from a given BM) the overall money supply:

$$M_s = C + D = \gamma D + D = (1+\gamma)D = \frac{1+\gamma}{\alpha+\beta+\gamma} BM = \frac{1+\gamma}{\alpha+\beta} BM_b \tag{3.5}$$

$$\Rightarrow \frac{dM_s}{dBM} = \frac{1+\gamma}{\alpha+\beta+\gamma}$$

$$\Rightarrow \frac{dM_s}{dBM_b} = \frac{1+\gamma}{\alpha+\beta}$$

Hence the M_s can be represented both as multiple of the BM or the BM_b, where the last to expressions represent their respective multipliers. We can arrive at this same relationships defining *high potential money* as $H = C + R$, where R are the bank reserves (precautionary or mandatory). Then, it follows that:

$$\begin{aligned} \frac{M_s}{H} &= \frac{C+D}{C+R} = \frac{(C+D)/D}{(C+R)/D} = \frac{1+c}{c+r} \\ \Rightarrow M_s &= \frac{1+c}{c+r} H = \frac{1+\gamma}{1+\alpha+\beta} H = mH \end{aligned} \tag{3.6}$$

Equation (3.6) implies that $c = \gamma$ and $r = \alpha + \beta$, hence the money multiplier $m = \frac{1+c}{c+r} = \frac{1+\gamma}{1+\alpha+\beta}$. **If money supply is assumed exogenous, then the CB's control this aggregate by varying the monetary base (high-potential money).** This last argument is based on two assumptions: (i) m is given, influenced by CB's or in any case foreseeable by them; and (ii) the actual credit disbursed in the economy is equal to the credit potentially disbursable by the banking system. Partially differentiating equation (3.6) with respect to H, α, β and γ we obtain the following relationships:

$$\frac{\partial M_s}{\partial H} = \frac{1+\gamma}{1+\alpha+\beta} > 0 \tag{3.7}$$

$$\frac{\partial M_s}{\partial \gamma} = \frac{H}{1+\alpha+\beta} > 0 \tag{3.8}$$

$$\frac{\partial M_s}{\partial \alpha} = -\frac{1+\gamma}{(1+\alpha+\beta)^2} H < 0 \tag{3.9}$$

$$\frac{\partial M_s}{\partial \beta} = -\frac{1+\gamma}{(1+\alpha+\beta)^2} H < 0 \tag{3.10}$$

Hence M_s increases with increases in the monetary base (or high-potential money H), with increases in the public's desire to keep currency in cash (γ), and with reductions in the proportions of discretionary (α) and mandatory (β) monetary base reserves that banks hold. It should be noted that Levrero wrote $\frac{\partial M_s}{\partial \gamma} < 0$, which is mathematically incorrect.

3.2 From Exogeneity to Endogeneity of Money

Some doubts regarding the capacity of CB's to control M_s: (i) changes in interest rates induce changes in the discretionary reserve ratio (α) and the propensity of the public to hold currency ()γ); and (ii) supply of credit adjusts to demand for credit, a lack of demand might imply excess reserves for private banks: hence they will a have a margin to vary supply of credit independently of the given BM set by the CB; (iii) H or BM is influenced by the economies demand for credit, given that CB's work as *lenders of last resort*: they do not limit total internal credit restricting the BM. Otherwise they would lose their ability to control interest rates.

Changes in M_s are obtained indirectly through the effect that changes in the interest rate set by CB's have on economic activity. We can then introduce a series of behavioral hypothesis regarding how agents in the economy react (mainly the public and banks) to changes in the interest rates on deposits (r_d), credit (r_c) and MRO's (r_m). Remembering that $M = C + D = \gamma D + D = (1+\gamma)D$, we can assume that:

$$BM_p = \gamma D = a_1 D - a_2 r_d \tag{3.11}$$

$$BM_b = (\alpha + \beta)D = (\beta + b_1)D + b_3 r_m - b_2 r_c \tag{3.12}$$

$$\Rightarrow BM = BM_p + BM_b = (a_1 + b_1 + \beta)D + b_3 r_m - a_2 r_d - b_2 r_c \tag{3.13}$$

$$\Rightarrow D = \left(\frac{1}{a_1 + b_1 + \beta}\right) BM + a_2 r_d + b_2 r_c - b_3 r_m \tag{3.14}$$

$$\text{With } \frac{\partial D}{\partial BM} > 0;\ \frac{\partial D}{\partial r_d} > 0;\ \frac{\partial D}{\partial r_c} > 0;\ \frac{\partial D}{\partial r_m} < 0 \tag{3.15}$$

Notice that if the MRO rate (r_m) goes up, then it becomes more expensive to obtain liquidity, which would imply a higher preference for banks to keep greater liquid reserves (and less deposits). **The supply of bank deposits (D) is no longer rigid, given BM this supply is influenced by portfolio adjustments made by the public and banks in light of changes in the various interest rates of the economy.** Hence, we can derive the following equation:

$$M_s = C + D = BM_p + D = a_1 D - a_2 r_d + D = (1 + a_1)D - a_2 rd$$

$$M_s = \left(\frac{1 + a_1}{a_1 + b_1 + \beta}\right) [BM + a_2 r_d + b_2 r_c - b_3 r_m] - a_2 r_d$$

$$M_s = \frac{1 + a_1}{a_1 + b_1 + \beta}[BM + b_2 r_c - b_3 r_m] + \frac{1 - b_1 - \beta}{a_1 + b_1 + \beta}(a_2 r_d) \tag{3.16}$$

$$\Rightarrow M_s = \phi(BM^+, r_d^+, r_c^+, r_m^-, \beta^-) \tag{3.17}$$

In equation (3.17) we can see that the money supply (M_s) varies positively through changes in the monetary base (BM) and the interest rates on deposits and credit (r_d and r_c, respectively). And, finally, that it varies negatively via changes in the rate of interest associated to MRO's (r_m) and the proportion of BM that banks have to hold as mandatory reserves (β).

In this approach the creation of money supply can be modified by the credit system in directions not necessarily desired by the CB, for example, an increase in r_m might be accompanied by an increase in r_c: hence monetary policy could have limited impacts on M_s. In the traditional view (the one regarding M_s as exogenous), that can be visualized in equation (3.6), CB's set M_s by directly controlling the BM: as long as M_d is stable we will find that changes in M_s will have definite-mechanical effects on interest rates, the price level and income in nominal terms.

If CB's could foresee changes in $m = f(r_d, r_c, r_m)$ such that $\Delta m \Rightarrow \Delta H$ in order for M_s to be constant ($\Delta M_s = 0$), then one could argue that M_s might show a certain *stability*, however that would still imply that CB's are somewhat *forced to adapt* the monetary base (BM) to changes in the money multiplier (m), determined in the private sector. **Two more elements can be brought up in order to argue in favor of an endogenous determination of M_s:** (i) potential disbursable credit might not be completely disbursed, excess reserves in the banking system may allow variations in the demand for loans and credit and bank deposits (given a BM); and (ii) CB's tend to adapt high-potential money to the credit needs of the economy.

The endogenous nature of money has been pointed out at least since the mid-nineteenth century, however, modern approaches also make emphasis in the fact that: (i) CB's tend to control the interest rate rather than M_s; (ii) CB's act as lenders of last resort; and (iii) the notion of money as liquidity rather than as a *unicum*. Modern theories on money endogeneity also argue that provided credit depends on the demand for loans and that the BM depends on the volume of credit. In fact:

1. CB's cannot reduce $\hat{M}_s$ because bank loans are determined by public demand (and cannot be purchased by the CB) and because the banking system, even in the light of changing interest rates, cannot reduce loans and credit lines already granted.

2. The demand for $BM = H$ is *rigid*, CB's only specify in what way and at what price it will be satisfied.

3. The banking system extends credit and then check weather their reserves are adequate for the purpose, if not, they request liquidity from the CB. The latter tends to fulfill such liquidity requests in orther to avoid tensions on interest rates.

4. MRO's of the banking system don't have quantitative limitations, these operations are used by CB's in order to push the banking system to use the *discount window*: with the aim of ensuring the CB's ability to control ST interest rates, and hence, the structure of interest rates.

5. CB's always leave their discount window open, it only affectd liquidity inderectly through the possible effect that changes in the interest rates may have on the demand for money and through this on the monetary aggregates that compose the supply of money.

6. If the CB doesn't behave in this way then:

   (a) We might expect strong fluctuations in the interest rate.

   (b) Creates a tendency towards financial innovation in the banking system (in order to fulfill their liquidity necessities), reducing the ability of the CB to control de structure of interest rates.

7. Changes in the mandatory monetary base requirements (β) will not necessarily have an effect on bank deposits (D), why? Because CB's will always provide the necessary liquidity in order to fulfill liquidity requirements.

8. Public's preference to hold currency, shifts in the balance of payments and discount requests of the banking system at the CB, are elements outside of the control of the latter: the maintenance of a *desired level* of BM only occurs in the presence of continuous compensation.

Hence, in modern monetary theories we have that $M_s$is endogenous because $H = BM$ (an its components) is endogenous. The variable controlled by the CB is the interest rate, given this rate the quantity of money is determined by the demand for money. Remembering that

$$M_d = kPY_r$$

$$M_s = M_d$$

$$M_sV = PY_r \tag{3.18}$$

It would follow that:(i) for Friedman changes in M_s would imply changes in the income in nominal terms via changes in the price level; and (ii) for Kaldor V would remain constant given that changes in PY_r entail changes in the nominal supply of money M_s. **If money supply is considered endogenous then changes in nominal income would no necessarily imply changes in the interest rate, and, of course, the constancy of *so called empirical* V could be understood as a consequence of the endogeneity of M_s rather than a ratification of the QTM.**

Kaldor argues that in Keynes's analysis of liquidity preference the assumed exogeneity of M_s would entail that adjustments in the money market, in the light of variations of real factors, are achieved via changes in V. This, of course, in the case in which M_s is considered independent of M_d. In a credit economy if M_s is higher than desired then this *surplus* would be extinguished via the repayment of debts or converting it into interest-bearing assets.

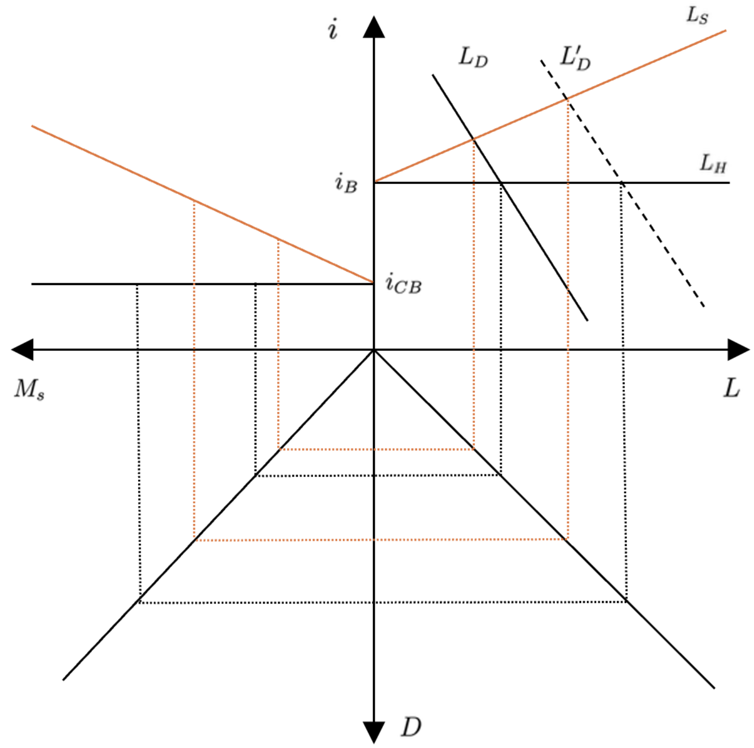

Figure 3.1: The debate between horizontalists (black) and *structuralists* (orange).

We can identify two schools of thought within post-keynesian economics regarding the nature of the relationship between the rate of interest and M_s. (i)

horizontalists believe that that M_s is horizontal with respect to the interest rate set by the CB; (*ii*) *structuralists* believe that the M_s is increasing with respect to the rate of interest, arguing that the credit supply curve (L_s) most be increasing with respect to the interest rate: only higher interest rates would push the banking system to grant greater amounts of credit (compensating the risk associated the *higher indebtedness* of the private sector). These two different positions can be observed in Figure 3.1. *structuralists* (orange)

Chapter 4

Money, Prices and Output in the Neoclassical Theory

In neoclassical theory the interest rate is the price that balances the demand and supply of savings, and it's determined in the savings-investment market: it reflects *productivity and thrift.* The interest rate is understood as a real phenomenon in the long-run. The analytical structure of the neoclassical theory can be summarized on some basic relations concerning the labor, goods and money markets.

We assume two factors of production N and K producing one commodity also used as K. Furthermore, the *data* of the theory is: (i) the given amount of $N = \bar{N}$ and $K = \bar{K}$; (ii) consumers preferences; and (iii) the technical conditions of production (the production function). In the labor market we have:

$$N_d = f(\frac{w}{P}) = f(w_r),\ \frac{dN_d}{dw_r} < 0 \tag{4.1}$$

$$N_s = g(\frac{w}{P}) = g(w_r),\ \frac{dN_s}{dw_r} > 0 \tag{4.2}$$

$$N_s = N_d = \bar{N} \tag{4.3}$$

Demand (supply) for labor is a decreasing (increasing) function of the real wage. Equation (4.1) represents the principle of substitution between factors of production (because the marginal rate of technical substitution is $MRTS_{KN} = \frac{w}{r}$). The third equation (4.3) represents the clearing condition of equilibrium. We also have a production function.

Which has the usual neoclassical properties: (i) constant returns to scale; (ii) decreasing marginal returns for K and L; and (iii) satisfies the *Inada Conditions*. It is defined as:

$$Y = F(K, N) \tag{4.4}$$

For a fixed $\bar{K}$, let Π be the profits function:

$$\Pi = YP - C = F(\bar{K}, N) - (wN + rK)$$
$$\max_L(\Pi) \Rightarrow MP_N P - w = 0 \Rightarrow MP_N = \frac{w}{P} = w_r$$

For a given w_r firms maximize profits employing the amount of N such that $MP_N = w_r$, so, given K these equation solve for a w_r such that $N_s = N_d$, the former being a reflection of the relative scarcity of N and K. Hence, in the presence of unemployment ($N_s > N_d$) competition in the labor market would ensure a fall in wages, and, consequently, the adoption of more labor intensive techniques.

However the tendency towards full employment needs the consideration of the savings-investment (or goods) market:

$$S = S(Y, r), \ \frac{dS}{dY} > 0 \text{ and } \frac{dS}{dr} > 0 \tag{4.5}$$
$$I = I(r), \ \frac{dI}{dr} < 0 \tag{4.6}$$
$$I = S \tag{4.7}$$

Equilibrium in this market, where households decisions to save coincide with investment decisions made by the firms, is achieved via variations in the interest rate: savings (investment) are an increasing (decreasing) function of r. The mechanism is the following

$$N_s > N_d \Rightarrow \downarrow w \Rightarrow \uparrow N \Rightarrow \uparrow Y \Rightarrow \uparrow S \Rightarrow S > I \Rightarrow \downarrow r \Rightarrow \uparrow I$$

The rate of interest r^* in which $S = I$, in correspondence to $Y = Y_p$ (Y_p being potential income), is called the *natural interest rate*: it depends on *productivity* and *thrift*. *Productivity* is related to I: investments are an inverse function of r because of decreasing marginal returns on capital, which affect the expected returns on investment (the lower the expected returns, the higher the amount of capital used). *Thrift* is related to

S: it stems from the constrained utility maximization problem of choosing between present and future consumption (savings). This occurs (the maximization) when $\frac{U'_t}{U'_{t+1}} - 1 = \rho = r = MP_K$, where ρ is the *marginal rate of intertemporal preference*, that is, the expectation of future consumption via the deferment of consuming a unit of consumption in the present. ρ is assumed to be: (i) positive (agents only give up consumption today if consumption tomorrow is going to be higher); and (ii) decreasing as present consumption increases, given that $U'_t < 0$. In equilibrium we will have that

$$\frac{U_t}{U_{t+1}} = 1 + \rho = 1 + r$$

If $\rho < r$, then agents would be induced to reduce present consumption C_t and increase future consumption C_{t+1}, increasing U'_t and reducing U'_{t+1} until $\rho = r$. From this it directly follows that: $(i) \uparrow r \Rightarrow \downarrow C_t \wedge \uparrow C_{t+1} \Rightarrow \uparrow \rho \Rightarrow \uparrow S$ (as long as the substitution effect prevails over the income effect); and $(ii) \uparrow Y \Rightarrow \uparrow C_t(\downarrow U'_t) \Rightarrow \rho < r$ (restarting the previous process described above). Hence $S = S(Y, r)$. Tanking the K and the production function as given, with equations (4.1)-(4.7) we determine Y, w_r, r, N_d, N_s, S and I: this is the equilibrium in real terms. The general prices is determined in the money market:

$$M_d = kPY_r \tag{4.8}$$

$$M_s^* = M_s = M_d \tag{4.9}$$

This is a QTM definition, since $M_s^* V = PY_r$ (remember that $v = \frac{1}{k}$). The exogenously fixed money supply determines de absolute price level given full employment real income Y^* and the stable velocity circulation of money. **The real side of the economy is solved separately from the monetary side: we have two self-contained subsystems of equations.** Hence, money is neutral: it doesn't affect the solutions of the real part of the model. **Summary**. In all neoclassical models we have that: (i) money is neutral, in the long-run changes in M_s do not affect the real variables (relative prices); (ii) there is a dichotomy between the real and monetary sectors.

4.1 Money in the Utility Function (MIU)

The model is constructed following this hypothesis: (i) $t = 1, 2, 3, ...$ (infinite and discrete time horizon); (ii) no uncertainty; (iii) no choice between labour and leisure; (iv) $z \sim m$ (the services stemming from per capital real liquid balances are proportional to the per capital real amount of money m); (v) agents utility is independent from period to period; (vi) at any time t we have that $u(c, m)$ is such that $u_c > 0$, $u_m > 0$, $u_{cc} < 0$ and $u_{mm} < 0$; (vii) $\lim_{m \to 0} u_m = \infty$; $(viii)$ $\forall c \; \exists \; m = \mu$ such that $u_m \leq 0 \forall m > \mu$, at equilibrium $m \in (0, \mu)$; and (ix) $Y_t = F(K_{t-1}, N_t)$, that satisfies the usual homogeneity of degree one (constant returns to scale, CRS), decreasing marginal productivity and the *Inada Conditions*.

For any time t we will have that $y_t \equiv \frac{Y_t}{N_t}$ and $k \equiv \frac{K_t}{N_t}$, using CRS, and remembering that $n = \frac{N_t}{N_{t-1}} - 1 \Rightarrow 1 + n = \frac{N_t}{N_{t-1}}$ we can write

$$y_t = F(\frac{K_{t-1}}{N_t} \cdot \left(\frac{N_{t-1}}{N_{t-1}}\right), 1) = f\left(\frac{k_{t-1}}{1+n}\right) \tag{4.10}$$

Where (i) $f_k \geq 0 \;\; \wedge f_{kk} \leq 0$; (ii) $\lim_{k \to 0} f_k = \infty$; and (iii) $\lim_{k \to \infty} f_k = 0$. The marginal product of per capita capital cannot become negative. The maximization problem, of intertemporal utility, of the representative agent which maximizes its total utility W is the following:

$$\max_{c_t, b_t, m_t} W = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t) \text{ s.t.} \tag{4.11}$$

$$\omega_t = f\left(\frac{k_{t-1}}{1+n}\right) + \tau_t + \left(\frac{1-\delta}{1+n}\right) k_{t-1} + \frac{(1 + i_{t-1} b_{t-1}) + m_{t-1}}{(1+n)(1+\pi_t)}$$

$$= c_t + k_t + m_t + b_t \tag{4.12}$$

Where β is the subjective discount rate. The constraint to be fulfilled at each time t is defined in equation (4.12). The given amount of resources (ω_t) must be equal to: per capita income (y_t), plus real per capita public net transfers (τ_t), plus per capita real value of net capital $(\frac{1-\delta}{1+n} k_{t-1})$, plus real per capita interests earned on bonds inherited from the past $(1 - i_{t-1}) \frac{B_{t-1}}{P_t N_t} (\frac{P_{t-1}}{P_{t-1}})(\frac{N_{t-1}}{N_{t-1}}) = \frac{(1-i_{t-1})b_{t-1}}{(1+n)(1+\pi)}$ and, finally, plus real per capita money balances inherited from the past $\frac{M_{t-1}}{P_t N_t}(\frac{P_{t-1}}{P_{t-1}})(\frac{N_{t-1}}{N_{t-1}}) = \frac{m_{t-1}}{(1+n)(1+\pi)}$. Where $b_t = \frac{B_t}{P_t N_t}$ and $m_t = \frac{M_t}{P_t N_t}$.

This constrained dynamic optimization can be solved by transforming it into an unconstrained maximization problem of an objective function V at time t which consists of the utility function and the optimal value of the same function V at time $t+1$. Knowing, by equation (4.12) that $k_t = \omega_t + c_t + m_t + b_t$ we have that the simultaneous optimization of current utility $u(c_t, m_t)$ and the value of resources at time $t+1$:

$$\begin{aligned}
\max V(\omega_t) &= \max_{c_t, b_t, m_t} \{u(c_t, m_t) + \beta V(\omega_{t+1})\} \\
&= \max_{c_t, b_t, m_t} \Bigg\{ u(c_t, m_t) + \beta V \Bigg[f\left(\frac{\omega_t + c_t + m_t + b_t}{1+n} \right) + \tau_t \\
&+ \frac{1-\delta}{1+n}(\omega_t + c_t + m_t + b_t) + \frac{(1+i_t)b_t + m_t}{(1+n)(1+\pi_{t+1})} \Bigg] \Bigg\} \qquad (4.13)
\end{aligned}$$

Letting $k^* = \frac{k_t}{1+n}$, we get the first order conditions for c_t, b_t and m_t and the *transversality conditions*, which are, respectively:

$$\begin{aligned}
&(a)\ \frac{\partial V(\omega_t)}{\partial c_t} = u_c - \frac{\beta}{1+n}(f_{k^*} + 1 - \delta)V_\omega(\omega_{t+1}) = 0 \\
&(b)\ \frac{\partial V(\omega_t)}{\partial b_t} = \frac{\beta}{1+n}\left[\frac{1+i_t}{1+\pi_{t+1}} - (f_{k^*} + 1 - \delta) \right] V_\omega(\omega_{t+1}) = 0 \\
&(c)\ \frac{\partial V(\omega_t)}{\partial m_t} = u_m - \frac{\beta}{1+n}\left[(f_{k^*} + 1 - \delta) - \frac{1}{1+\pi_{t+1}} \right] V_\omega(\omega_{t+1}) = 0 \\
&(d)\ \lim_{t\to\infty} \beta^t x_t u_c = 0, \text{ for } x = k_t, b_t, m_t
\end{aligned}$$

We obtain three key results from these past four conditions. (*I*) the *fisher equation*, using (b) and (d). Let $r = f_{k^*} - \delta$ be the net marginal product of capital and let $\pi_{t+1} = \pi^e$ be the expected inflation rate. It is also evident that $\frac{1+i_t}{1+\pi_{t+1}} - [f_{k^*} + 1 - \delta] = 0$ (since $\frac{\beta}{1+n} > 0$ and $V_\omega(\omega_{t+1}) > 0$) hence we have that:

$$\begin{aligned}
&\frac{1+i_t}{1+\pi^e} - (1+r) = 0 \\
&\Rightarrow (1+i_t) = (1+r)(1+\pi^e) \\
&1 + i_t = 1 + r + \pi^e + r\pi^e \\
&\text{If } \ r\pi^e \approx 0 \Rightarrow r = i - \pi^e \qquad (4.14)
\end{aligned}$$

(II) Using (a), (c) and (d), we get that **the marginal gain due to an increase in the money balances m at time t must be equal to the marginal utility of consumption at time t**:

$$u_m + \frac{\beta u_{c_{t+1}}}{(1+\pi_{t+1})(1+n)} = u_c \qquad (4.15)$$

The marginal utility gain can be divided into two parts. Where u_m is the *direct increase in utility* stemming from the additional amount of per capita money balances at t, and where $\frac{\beta u_{c_{t+1}}}{(1+\pi_{t+1})(1+n)}$ is the discounted marginal utility of consumption at time $t+1$ (this is the *indirect increase in utility*): given by the fact that additional money in t yields in an increase in wealth of $\frac{m_t}{(1+\pi_{t+1})(1+n)}$ at time $t+1$, wealth that can be utilized to buy consumption goods at $t+1$ (consumption that yields an specific marginal utility $u_{c_{t+1}}$).

(III) Using (a), (b) and equation (4.15) we obtain the following result: **the the marginal rate of substitution between m and c must be equal to the opportunity cost of holding money, that is, the discounted value of the interest rate:**

$$\frac{u_m}{u_c} = \frac{i_t}{1+i_t} = \frac{i}{(1+r)(1+\pi^e)} \tag{4.16}$$

The relative price of m in terms of c is directly linked to the nominal rate of interest i. Hence, given r, the higher the inflation expectations (π^e) are, the lower the real value of money will be.

4.1.1 The MIU Model in Strict Stationary State Conditions

In Solow's growth model we have a situation in which, through substitution between labor and capital, $g_n = g_w = \frac{s}{v}$, where s is the marginal propensity to save and $v = \frac{K}{Y}$ is the capital output ratio. Following the Harrod-Domar model notation we can see that:

$$\Rightarrow g_w = \frac{s}{v_b} < g_n = \frac{s}{v_a} \Rightarrow L^d < L^s \Rightarrow\downarrow w \Rightarrow k_b \to k_a(v_b \to v_a) \Rightarrow g_w = g_n = \frac{s}{v_a}$$
$$g_w = \frac{s}{v_a} > g_n = \frac{s}{v_b} \Rightarrow L^d > L^s \Rightarrow\uparrow w \Rightarrow k_a \to k_b(v_a \to v_b) \Rightarrow g_w = g_n = \frac{s}{v_b}$$

In a situation where there is no technical progress we have that $g_n = n = \frac{\dot{L}}{L}$. The adjustments of the warranted growth rate towards the natural growth rate are achieve through changes in v. Let $n = 0$, which is a *strict stationary state* different from the canonical one described in Solow's growth model, and also let $\frac{\dot{M}_s}{M_s} = \vartheta = \pi_{ss} = \pi^e$: in an stationary state the inflation rate is equal to the growth rate of the money supply, which we name ϑ. Also, note that $\omega_t = \omega_{t+1}$ (resources don't change through time).

Then the maximization problem takes the following form, where the subscript s indicates that the variable is on its *stationary state path*:

$$\begin{aligned}\max V(\omega) &= \max_{c_s,b_s,m_s} \{u(c_s,m_s)+\beta V(\omega)\} \\ &= \max_{c_s,b_s,m_s} \Bigg\{u(c_s,m_s)+\beta V\Bigg[f(\omega_s+c_s+m_s+b_s)+\tau_s \\ &+1-\delta(\omega_s+c_s+m_s+b_s)+\frac{(1+i_s)b_s+m_s}{1+\vartheta}\Bigg]\Bigg\}\end{aligned} \tag{4.17}$$

The first order conditions for c_s, b_s and m_s are, respectively:

$$\frac{\partial V(\omega)}{\partial c_s} = u_{c_s} - \beta(f_{k_s}+1-\delta)u_{c_s} = 0 \tag{4.18}$$

$$\frac{\partial V(\omega)}{\partial b_s} = \frac{1+i_s}{1+\vartheta} - (f_{k_s}+1-\delta) = 0 \tag{4.19}$$

$$\frac{\partial V(\omega)}{\partial m_s} = u_{m_s} - \beta(f_{k_s}+1-\delta)u_{c_s} + \frac{\beta u_c}{1+\vartheta} = 0 \tag{4.20}$$

Here we have that $m = \frac{M_s}{P} \Rightarrow$ money appears in real terms, hence, ***money is neutral*** as long as we don't find ourselves in *money illusion* situation (that is, a situation in which changes in the level of prices alter the demand for money in real terms): changes in M_s leave $M_{sr} = \frac{M_s}{P}$ unchanged. This $\frac{\dot{M_{sr}}}{M_{sr}} = \vartheta - \pi_{ss} = \vartheta - \vartheta = 0$. To observe that in the MIU model that we also have ***super neutrality of money*** we need to show that changes in M_s do not have any effect on the real variables y_s, c_s and k_s. Remember that, in a Cobb-Douglas technology (without technical change), we have that $y = f(k) = k^\alpha$ such that $f_k = \alpha k^{\alpha-1}$. Dividing both sides of equation (4.18) by u_{c_s} we obtain:

$$f_{k_s} = \frac{1}{\beta} - 1 + \delta \tag{4.21}$$

$$\text{In a Cobb-Douglas technology } \Rightarrow f_{k_s} = \alpha k^{\alpha-1} = \frac{1}{\beta} - 1 + \delta$$

$$\Rightarrow k_s = \left[\frac{\alpha\beta}{1+\beta(\delta-1)}\right]^{\frac{1}{1-\alpha}} \tag{4.22}$$

The stationary state capital-labor ratio (capital intensity) ratio is completely independent of the rate of inflation ϑ and any parameter of the utility function. It depends only on α (the elasticity of output with respect to capital), δ (depreciation rate) and β (subjective discount rate). With respect to consumption, manipulating the budget constraint (setting $b_s = 0$ and $\tau_s = \frac{\vartheta m_s}{1+\vartheta}$) $f(k_s) + \tau + (1+\delta)k + \frac{m_s}{1+\vartheta} = c_s + k_s + m_s$ we have that $c = f(k_s) - \delta k_s$. Using equation (4.22), with a Cobb-Douglas technology, we will have that consumption at the stationary state will be given by the following equation:

$$c_s = k_s^\alpha - \delta k_s = \left[\frac{\alpha\beta}{1+\beta(\delta-1)}\right]^{\frac{\alpha}{1-\alpha}} - \delta\left[\frac{\alpha\beta}{1+\beta(\delta-1)}\right]^{\frac{1}{1-\alpha}} \tag{4.23}$$

Again, we have that consumption per capita, which is a real variable, depends only only on α (the elasticity of output with respect to capital), δ (depreciation rate) and β (subjective discount rate): **changes in M_s (and hence in the general level of prices) have no effect in consumption.** Moreover, note that since $c_s = c_t = c_{t+1}$ and $m_s = m_t = m_{t+1}$, equation (4.21) can actually be written as:

$$1 = \frac{u_{c_t}}{u_{c_{t+1}}} = \beta(f_{k_s} + 1 - \delta)$$

$$\frac{1}{\beta} = (f_{k_s} + 1 - \delta) \tag{4.24}$$

If $k < k_s \Rightarrow f_{k_s} < f_k \Rightarrow \frac{1}{\beta} < (f_k + 1 - \delta) \Rightarrow$ consumption is postponed $\Rightarrow \uparrow k \Rightarrow f_k \rightarrow f_{k_s}$. The same mechanism applies for a situation in which $k_s < k$. Also, note that, if $\uparrow \theta \Rightarrow \uparrow k \Rightarrow (f_k + 1 - \delta) < \frac{1}{\beta} \Rightarrow$ increase in present consumption $\Rightarrow \downarrow k \Rightarrow f_k \rightarrow f_{k_s}$. **A change in ϑ only leads to a change in the inflation rate, and by equation (4.14), it also has an effect in the nominal rate of interest.** In the stationary state c_s and m_s are such that:

$$\frac{u_{m_s}}{u_{c_s}} = \frac{i_s}{1+i_s} \tag{4.25}$$

(**Existence**) If at any time t, we have that utility function $u(c,m)$ can be *separated*: $u(c,m) = v(c) + h(m) \Rightarrow h_m = \frac{i_s}{1+i_s} v_c$. We have that $\frac{i_s}{1+i_s} v_c > 0 \iff i_s \neq 0 \wedge v_c \neq 0$, and that $\lim_{m \to 0} h_m = \infty \wedge h_m < 0 \forall m > \mu$. Under this conditions equilibrium exists: (i) there will be a positive demand for m; (ii) if $h_m < 0 \Rightarrow \Delta m < 0 \wedge \Delta c > 0 \Rightarrow \Delta h_m > 0 \wedge \Delta \frac{i_s}{1+i_s} v_s < 0$. (**Uniqueness**) We have a situation of *multiple equilibria* when: (i) $u(c,m)$ is not separable at each t; and (ii) if $\Delta m > 0 \Rightarrow \Delta u_c < 0$. (**Stability**) If the transversality condition is satisfied (d), then the equilibrium is stable.

4.2 The Optimum Quantity of Money

The private opportunity cost (PMC) of holding money is the nominal interest rate, in a fiat-money economy, the social marginal cost of producing money (SMC) is zero. When

$i > 0 \Rightarrow SMC < PMC \Rightarrow$ the inefficiency is eliminated only when $i = 0$. Following equation (4.14), if $i = 0 \Rightarrow \pi^e = \left[\frac{1}{(1-r)}\right] - 1 = \frac{-r}{(1-r)} \approx -r$: **the optimum rate of inflation is a deflation rate equal to the rate of profit.** Given that $g_n = n$, the optimum growth rate of M_s is $\vartheta - n = \pi = -r$. As in seen in Subsection 4.1.1, along the steady state path $\vartheta = \pi$, which implies no effects on the real quantity of money balances per capita (m), and thus no effect on the agents utility.

If $n = 0 \Rightarrow \max u(c_s, m_s)$ and the net saving are equal to zero $\Rightarrow c_s = f(k_s) - \delta k_s$. We already know that c_s and u'_{c_s} are independent of ϑ, by equations (4.16) and (4.25) we can see that $\frac{\partial m}{\partial \vartheta} = 0 \iff i = 0$. This can be criticized on the basis that, it might be the case that if $\uparrow M_s \Rightarrow \uparrow T$. Friedman also recognizes that *lags* in the adjustment of nominal prices imply the necessity of a low, but positive, rate of inflation (which implies a positive nominal money growth rate).

4.3 Overlapping Generations Model

Consider a standard endowment economy with two-period agents and one good. If $n = 0 \Rightarrow R_t = \frac{1}{(1+i_t)} = 1$, where R_t is the discount rate between goods from period t and goods from period $t+1$. The absence of money implies that, if $n > 0 \Rightarrow c_t = e_1 \wedge c_{t+1} = 0$ (if the good is perishable). The latter situation is not *pareto optimal*, any transfer of agents in t to agents in $t+1$ would be a *pareto improvement*. With the existence of money, we would have that if p_t is the amount of money required to buy one unit of goods at t, then, the return on money would be $\frac{p_t}{p_{t+1}} = 1 + r$ ($\frac{p_t}{p_{t+1}} - 1 = r$). If $\vartheta < n \Rightarrow r_{t+1} > 0$.

The assumptions of the model are: (i) $t = 1, 2, 3, ...$ (discrete infinite horizon); (ii) agents live two periods; (iii) in any period t a new generation substitutes the *old people* of the previous period; (iv) this *new generation* has an initial endowment of a homogeneous and perishable consumption good. The utility function is

$$u = (c_t, c_{t+1}) \tag{4.26}$$

As previously mentioned, in a *moneyless* economy there is no incentive towards inter-generational exchange because: (i) agents give in t to receive in $t+1$; (ii) agents receiving in

$t+1$ cannot give anything in $t+2$; (*iii*) agents willing to give in t will not appear until $t+1$. In this case *autarky* is the only possible equilibrium. With money: old agents in t have an incentive to exchange money for consumption goods; young people in t have an incentive to accept this money in order to consume in $t+1$. **Money, not being perishable, is then both a medium of exchange and a reserve of value.** The maximization problem is:

$$\max_{c_t, c_{t+1}} u(c_t, c_{t+1}) \text{ s.t.} \tag{4.27}$$

$$p_t c_t + m_t = p_t \tag{4.28}$$

$$\text{and } p_{t+1} c_{t+1} = m_t \tag{4.29}$$

We can substitute equation (4.29) in equation (4.28) to get: $p_t c_t + p_{t+1} c_{t+1} = p_t \Rightarrow c_t + \frac{1}{1+r} c_{t+1} = 1$. We can now write the following maximization problem with its respective *lagrangian operator*:

$$\begin{aligned}
&\max_{c_t, c_{t+1}} u(c_t, c_{t+1}) \text{ s.t. } 1 - c_t - \frac{1}{1+r} c_{t+1} = 0 \\
&\mathcal{L}(c_t, c_{t+1}, \lambda) = u(c_t, c_{t+1}) + \lambda \left[1 - c_t - \frac{1}{1+r} c_{t+1} \right] \qquad (4.30) \\
&\frac{\partial \mathcal{L}}{\partial c_t} = u_{c_t} - \lambda = 0 \Rightarrow \lambda = u_{c_t} \\
&\frac{\partial \mathcal{L}}{\partial c_{t+1}} = u_{c_{t+1}} - \frac{\lambda}{1+r} = 0 \Rightarrow \lambda = u_{c_{t+1}}(1+r) \\
\Rightarrow &\frac{u_{c_t}}{u_{c_{t+1}}} = (1+r) = \frac{p_t}{p_{t+1}} \qquad (4.31)
\end{aligned}$$

Equation (4.31) means that the marginal rate of substitution between present consumption c_t and future consumption c_{t+1} is equal to $1+r \Rightarrow \uparrow \pi \Rightarrow \downarrow u_{c_t} \wedge \uparrow u_{c_{t+1}} \Rightarrow \uparrow c_t > 0$. Some problems outlined in the literature about this solution to introduce money into the general equilibrium framework: (*i*) the equilibrium is not unique, it's affected by the inflation rate and can be indeterminate in an infinite horizon; (*ii*) inflation implies the that old would consume *few*; (*iii*) money might be valuable today only because is going to be valuable tomorrow: money is a bubble without intrinsic value, its accepted because of those expectation; (*iv*) in a non-monetary economy giving money to people would result in a dynamically efficient steady state, however if the economy is dynamically inefficient without money, money will never be valued in equilibrium; and (*v*) money would be dominated by any asset that provides an interest rate.

4.4 Short/Medium-run Output Fluctuations

In neoclassical theory in the long-run (under price flexibility and fulfillment of expectations), economies tend towards a full employment growth path. **Fluctuations in the short-run are due to:** (i) money illusion (that is, a situation in which changes in the level of prices alter the demand for money in real terms); (ii) price rigidities and adjustment lags; and (iii) forecast errors.

Following fisher, nominal output can fluctuate due to monetary disturbances, under the assumption of adaptive expectations. At equilibrium we have that $i = r + \pi^e = r_n + \vartheta$. That is, the real interest rate is equal to the *normal interest rate* associated to *productivity and thrift*. Then we can see that

$$\uparrow \vartheta \Rightarrow \uparrow \frac{M}{P} \Rightarrow \downarrow i \Rightarrow \downarrow r \Rightarrow r < r_n$$

However, if $r < r_n \Rightarrow$

$$AD > AS(Y > Y_p) \Rightarrow \uparrow \pi^e \Rightarrow \uparrow i \Rightarrow r \rightarrow r_n$$

If $\pi > \vartheta \Rightarrow \downarrow \frac{M}{P}, \uparrow i \Rightarrow r \rightarrow r_n \Rightarrow Y \rightarrow Y_p$. Similar to Fisher, Wicksell presented an analysis in which: (i) there is money endogeneity; (ii) there is a criticism of the QTM; and (iii) there is a distinction between *market* and *natural* interest rates. As before: r_n is determined by *productivity and thrift*. The existence of a banking system implies that the supply of loans is not limited by savings: $i \neq r_n$. We have two scenarios:

$$(i) i < r_n \Rightarrow \uparrow AD \text{ if } Y_r = Y_p \Rightarrow \uparrow P \Rightarrow \textit{ cumulative phase of inflation}$$

$$(ii) i > r_n \Rightarrow \downarrow AD \text{ if } Y_r = Y_p \Rightarrow \downarrow P \Rightarrow \textit{ cumulative phase of deflation}$$

Eventually there is pressure on the banking system to adjust $i \rightarrow r_n$, why? Wicksell is not clear about this: sometimes he refers to the possibility of a pressure on the bank reserves due to an increase in bank deposits. In the presence of wage and price rigidities there is the possibility of real fluctuations. Also, there is the discussion regarding forced savings: $\uparrow P \Rightarrow \downarrow C \Rightarrow \uparrow S \Rightarrow$ **this would imply that r_n is adjusting towards i due to capital accumulation.** For Wicksell, there is not a relevant change in the capital stock, hence, this are temporary effects.

Chapter 5

Keynes, IS-LM and Expectations

5.1 From the *Treatise on Money* to the *General Theory of Employment, Interest and Money*

In the *Treatise on Money* Keynes: (i) lays out a critique of the QTM; (ii) makes a comparison between i and r_n; and (iii) analyzes the possibility of forced savings. As in the *Cambridge Equation Model* savings adjust to investments through changes in income distribution: $s_c > s_w \approx 0$ (Keynes, 2011).

In the *General Theory*, Keynes introduced two innovative elements of analysis: (i) the principle of effective demand: $I \Rightarrow S$, through changes in the level of income given money wages; (ii) the rate of interest as a *monetary phenomenon*, it is not the variable that ensures that $S \Rightarrow I$: it is, rather, determined in the money market by M_s and M_d. However changes in i and not in P ensure $M_s = M_s$, in contrast to the QTM.

We can write the essential points of the principle of effective demand by a simple model. Consider a closed economy with state intervention, the economy can be characterized by the following system of equations. We have the equilibrium between aggregate supply $Y = C + S + T$ and aggregate demand $Z = C + I + G$:

$$Y = Z = C + I + G = C + S + T \Rightarrow S + T = G + I \tag{5.1}$$

We have keynesian consumption function, and the definition of *disposable income* and taxes. We can then fin the following expression for consumption:

$$C = c_0 + c_1 Y_d \tag{5.2}$$

$$Y_d = Y - T \tag{5.3}$$

$$T = t_0 + t_1 Y \tag{5.4}$$

$$\Rightarrow C = c_0 + c_1(1 - t_1)Y - c_1 t_0 \tag{5.5}$$

Finally, we have that $G = G_0$ and $I = I_0$, hence exogenous-autonomous. We can define the equilibrium as:

$$Y^* = \frac{1}{1 - c(1 - t_1)}\Big[c_0 - c_1 t_0 + I_0 + G_0\Big] = m(c^+, t_1^-) \cdot A \tag{5.6}$$

We from equation (5.6), can see that **there is no mechanism that guarantees $Y^* = Y^p$, where Y^p is potential output: from this simple model stems the possibility of under-employment equilibria** (in the presence of a lack of aggregate demand, and hence the necessity of state intervention). State intervention such as: (*i*) accommodative monetary policies to minimize the cost of debt payment; (*ii*) deficit spending and socialization of a share of investments; (*iii*) the *euthanasia of rentiers* and redistribution measures; (*iv*) income policies for price stabilization.

In this analytical framework there is no *a-priori* method of expressing every economic interaction in mathematical terms: thus the relevance of social, political, institutional and historical factors. From this *linear* way of reasoning:

$$M_d = M_s \Rightarrow i \Rightarrow I(i, j) \Rightarrow Y \Leftarrow m, G$$

Where j is the exogenously determined marginal efficiency of capital. In this last example there is not a simultaneous determination fo equilibrium in the money and goods markets: the results change according to contextual circumstances. In the Keynes's *General Theory* we have traditional (neoclassical) and innovative elements. From the traditional point of view, it retains: decreasing marginal product of labor and decreasing marginal efficiency of capital, and an inverse relationship between investments and the rate of interest (there is and $i \Rightarrow S = I$).

5.2 Fiscal and Monetary policy in the IS-LM

In this model the equilibrium between the money market (LM) and the goods market (IS) is determined simultaneously. The equilibrium in the commodity market can be written by the following system of equations:

$$S - I \equiv G - T \tag{5.7}$$

$$Y = C + G + I \tag{5.8}$$

$$C = c_0 + c_1 Y_d \tag{5.9}$$

$$Y_d = Y - T \tag{5.10}$$

$$T = t_0 + t_1 Y \tag{5.11}$$

$$I = I_0 - ar + vY_d \tag{5.12}$$

$$E = c_0 + I_0 + G - (c+v)t_0 \tag{5.13}$$

$$m = \frac{1}{1-(c+v)(1-t)} \tag{5.14}$$

$$\Rightarrow Y = \frac{c_0 + I_0 + G - (c+v)t_0}{1-(c+v)(1-t_1)} - \frac{a}{1-(c+v)(1-t_1)} r = m(E - ar) \tag{5.15}$$

$$\Rightarrow r = \frac{1}{a}[c_0 + I_0 + G - (c+v)t_0] - \frac{1-(c+v)(1-t)}{a} Y = \frac{E}{a} - \frac{1}{am} Y \tag{5.16}$$

The IS curve is the pairs of income and interest such that the equality in equation (5.7) is fulfilled. Note that if $1 > (c_1 + v)(1 - t_1) \Rightarrow$ the IS curve is decreasing. With regards to the money market we have:

$$M_d = kY + l_0 - hr \tag{5.17}$$

$$M_s = \bar{M} \tag{5.18}$$

$$M_s = M_d \tag{5.19}$$

$$\Rightarrow r = \frac{k}{h} Y + \frac{1}{h}(l_0 - \bar{M}) \tag{5.20}$$

We can introduce equation (5.20) into equation (5.15) and obtain the following expression:

$$Y = \frac{1}{1-(c+v)(1-t_1) + \frac{ak}{h}} [c_0 + I_0 + G + \frac{a}{h}(\bar{M} - l_0) - (c+v)t_0] \tag{5.21}$$

This last equation (5.21) is the IS-LM model in reduced form. We can use this expression to obtain the public expenditure multiplier and the monetary multiplier:

$$m^G = \frac{\partial Y}{\partial G} = \frac{1}{1-(c+v)(1-t)+\frac{ak}{h}} = \frac{mh}{h+a} \tag{5.22}$$

$$m^m = \frac{\partial Y}{\partial M} = \frac{\frac{a}{h}}{1-(c+v)(1-t)+\frac{ak}{h}} = \frac{am}{h+am} \tag{5.23}$$

We can see that $\uparrow a \Rightarrow \uparrow m^m \wedge \downarrow m^G$, and that $\lim_{h\to\infty} m^G = m$ and $\lim_{h\to\infty} m^m = 0$. Remember that h is a parameter that associates negatively M_d with r, such that is related to the *speculative demand for money* and a is the parameter that associates the sensitivity of investment to the rate of interest. Note that the *monetary retraction* is measured, in both equations (5.22-5.23) by $\frac{ak}{h}$. **If $h = 0 \Rightarrow$ fiscal policy is ineffective and monetary policy is effective, if $h = \infty$ (liquidity trap) then fiscal policy is effective and monetary policy is ineffective.**

5.3 The Role of Expectations

In the IS-LM model there exists a certain M such that $u = \frac{N-L}{L} = u^t$ is equal to the one targeted by public decision makers. If $\uparrow AD \Rightarrow \uparrow Y \wedge \downarrow u \wedge \Delta M_s = 0 \Rightarrow \uparrow P \Rightarrow \uparrow AS \Rightarrow \downarrow Y \Rightarrow \uparrow u$. **If policy makers want to maintain the unemployment rate resulting from the AD positive shock, then $\uparrow M_s$ such that the rate of interest remains unchanged.**

(i) During a crisis, in the presence of liquidity trap conditions ($h \approx \infty$) and an inelastic investment-interest relationship ($a \approx 0$), then expansionary fiscal policies are useful. (ii) In *normal times*, automatic stabilizers and monetary policies measures accomplish target specific variable levels regarding: inflation, unemployment, balance of payments equilibrium, etc. This under the framework in which expansionary fiscal policies lead to crowding out: which has an effect on capital accumulation and thus on the trend of potential output and productivity.

Regarding the debate with the monetarists: (*i.1*) the *Keynes effect*, which is an indirect effect passing through the effects of a change in the rate of interest on aggregate demand; (*i.2*) the *Pigou effect*, this is a direct effect on consumption due to an increase in the amount of real money balances, or, during a crisis also due to deflation. (*ii*) There is also a debate on the value of fiscal and monetary multipliers, for the monetarists $m^m > m^g$ (vertical LM curve, expansionary fiscal policies are ineffective). Also, it is assumed in this framework that consumption depends only on *permanent income*, such that transitory effects on observed income would only have an effect on savings. (*iii*) The revision of expectations: augmented Phillips curve (dependent on expected inflation, expectations are subject to revisions); a long run vertical Phillips Curve. According to monetarists you also have a long-run vertical Phillips curve for the case of adaptive expectations, and, in the case of the *New Classical Macroeconomics* there is also a short-run vertical Phillips curve in the case of rational expectations.

Expectations are not absent in Keynes's analysis. We distinguish three main *expectations schemes*:

$$(i)\ x_t^* = x_{t-1} + \lambda(x_{t-1} - x_{t-2}) \text{ (extrapolative)}$$

$$(ii)\ x_t^* = x_{t-1}^* + \lambda(x_{t-1} - x_{t-1}^*) \text{ (adaptive)}$$

$$(iii)\ p_t^* = \frac{a-c}{b+d} - \frac{1}{b+a}E(u_t) \text{ if } E(u_t) = 0 \Rightarrow p_t^* = \frac{a-c}{b+d} \text{ (rational)}$$

In (*i*) we have that the expected value is the previous value plus a correction on the base of it's discrepancy with the value of two previous years. In (*ii*) the adjustment is made by the forecast error in the previous year, where $x_t^* = \lambda \sum_{n=1}^{t}(1-\lambda)^{n-1}x_{t-n}$ is weighted average of the past values where a greater weight is ascribed to more recent values. (*iii*) Is the rational expectation example made by Muth: it is a conscious forecast according to a *relevant-true* economic model; the scarcity of information implies its efficient use; they stem from an endogenous expectations model. In this example the expected value of the price of a commodity is a function, not from past values, but from all the information and the structural parameters of the model. **Consequences of the introduction of rational expectations within the traditional framework:** fine tuning is destabilizing; fixed rules for monetary policy; and the ineffectiveness of economic policies.

5.4 Lucas and the New Classical Macroeconomics

In this framework: (i) monetary and fiscal policies are ineffective both in the short and long-run unless agents are *surprised*; and (ii) economic policies are optimal when they are *supply-side* policies (reduction of frictions and rigidities in the labor and commodity markets). The AD-AS model with rational expectations is:

$$Y_t^d = a - bP_t = aZ_t - bP_t \tag{5.24}$$

$$Y_t^s = Y_o + d(P_t - P_t^*) + u_t \tag{5.25}$$

$$Y_t^d = Y_t^s \tag{5.26}$$

$$P_t^* = E\left(\frac{P_t}{I_{t-1}}\right) \tag{5.27}$$

Where Y_t^d is AD and Z_t is variable that includes the instruments of *economic policy* (fiscal and monetary). **Where Y_t^s is AS and $E(u_t) = 0 \Rightarrow$ output (Y_t) diverges from potential output (Y_o) only in the presence of stochastic shocks and differences between observed and expected prices.** Solving with respect to P_t, and substituting that result into equation (5.24):

$$\begin{aligned} P_t &= \frac{aZ_t + dP_t^* - Y_o - u_t}{b+d} \\ \Rightarrow Y_t &= \frac{adZ_t - bdP_t^* + bY_o + bu_t}{b+d} \end{aligned} \tag{5.28}$$

From the last equation we can see that $Y_t = f(\overbrace{Z_t}^{+}, \overbrace{P_t^*}^{-}, ...)$, so, in fact, regardless of accidental supply shocks, we have that $\uparrow Z_t \Rightarrow \uparrow P_t \Rightarrow$ forecast errors $\Rightarrow$ workers increasing their supply of hours worked and firms using their plants beyond their normal utilization. However, expectations are adjusted since we can derive the following expression:

$$P_t^* = \frac{aE(Z_t) - Y_o}{b} \tag{5.29}$$

This last expression implies that the expected price level depends on the expected economic policy. We can introduce this result on equation (5.28):

$$Y_t = \frac{ad[Z_t - E(Z_t)]}{b+d} + Y_o + \frac{bu_t}{b+d} \tag{5.30}$$

Therefore, any gap between Y_t and Y_o stems from differences between Z_t and $E(Z_t)$: economic policy is only effective if it diverges systematically from the expected economic policy. For Friedman it can occur in order to avoid output fluctuations (fixed rules), for Lucas, in the presence of rational expectations, there is no systemic divergence. If $Z_t - E(Z_t) = \varepsilon_t$, such that there is a systematic relationship for economic policies, then we have that:

$$Y_t - Y_o = \frac{ad\varepsilon_t + bu_t}{b+d} \tag{5.31}$$

Hence: (i) the output gap is only explained due to random factors independent of the policy rule; and (ii) fiscal and monetary policies are impotent. However: (a) different policies have different random errors and this a difference in variance of income; (b) the ineffectiveness of fiscal and monetary policies disappears in the case of adaptive expectations; and (c) if expectations are rational but the supply curve is not vertical in the long-run, we get a situation in which policies have effects on real output.

In the presence of adaptive expectations where $P_t^* = P_{t-1}^* + \lambda(P_{t-1} - P_{t-1}^*)$ we have that:

$$Y_t = \frac{adZ_t - bd\lambda P_{t-1} - bd(1-\lambda)P_{t-1}^* + bY_o + bu_t}{b+d} \tag{5.32}$$

In all t only Z_t is an unknown, therefore $Y_t = f(Z_t^+, ...)$: output is directly related to Z_t, economic policies are effective. Still assuming rational expectations but wage rigidity, such that $Y_t^s = c + dP_t^* + u_t$, we get:

$$P_t^* = \frac{a}{b}E(Z_t) - \frac{c}{b+d}$$

$$Y_t = dP_t^* + c + u_t$$

$$\Rightarrow Y_t = \frac{ad}{b}E(Z_t) + \frac{cb}{b+d} + u_t \tag{5.33}$$

This last equation implies that economic policy has an effect on output. If there is a monetarist or fiscal, which is followed, with the form $M_t = kM_{t-1} + \varepsilon_t$ or $G_t = kY_{t-1} + \varepsilon_t$, respectively, then the rule still has an effect on output. From this we can see that: (i) even within the same theoretical approach different schemes of expectations formation lead to different solutions; and (ii) it's not to much the chosen scheme of expectations the one that differentiates different theoretical approaches, but rather the different structure of the models.

Chapter 6

The New Keynesian and Post-Keynesian Theories

6.1 The New Keynesian Model

The main difference between New Classical Macroeconomics and Real Business Cycle Models (RBC) is the assumption of an endogenous money supply in the latter one: the authorities set the interest rate at the natural level in order to avoid *cumulative phases* of inflation or deflation. **The main features of RBC models:** (i) rational expectations micro-foundations: agents dynamically maximize utility over a certain time horizon; (ii) the existence of *equilibrium cycles* (there is no disequilibrium); and (iii) the prescription of supply-side economic policies to reduce *imperfections*. However, as seen before, even with rational expectations, in the presence of price rigidities, demand shocks have real effects: economic policies have an impact on real output. This is the starting point of New Keynesian Models (NK).

The NK synthesis: (i) rigidities in the labor and commodities markets (difference with RBC): prices form via the *Calvo rule* (firms adjust prices according to cost changes), there is monopolistic competition, there is money wage rigidity; (ii) intertemporal utility maximization (agents are *forward looking*); (iii) money endogeneity. The NK model can be summarized by the following system of equations:

$$\pi_t = \beta E(\pi_{t+1}) + kx_t + \lambda_t \text{ (non-vertical Phillips curve)} \tag{6.1}$$

$$x_t = E(x_{t+1}) - \sigma\{[i_t - E(\pi_{t+1})] - r_t^n\} \text{ (dynamic IS curve)} \tag{6.2}$$

$$r_t^n = E(cg_{t+1}) + E(z_{t+1}) \text{ (the natural rate of interest)} \tag{6.3}$$

$$i_t = r_t^n + E(\pi_{t+1}) + a_\pi(\pi_t - \pi^T) + a_y(y_t - y^p) \tag{6.4}$$

Where $x_t = y_t - y^p$ is the output gap; π_t is the inflation rate; π^T is the *target inflation rate*; λ_t is parameter associated to shocks on desired mark-ups in the commodities and labor markets; σ is the households intertemporal elasticity of substitution between present and future consumption; r_t^n is the natural interest rate; g_t is the natural growth rate of output; z_t is a variable containing all other factors influencing the natural rate of interest (including the rate of temporal preference); and, finally, k is the reactivity of inflation to changes in the output gap.

In equation (6.2) we have: (i) the usual inverse relationship between the rate of interest and aggregate demand; (ii) an expected future consumption greater than the normal one implies a higher present consumption; (iii) x_t depends on the current and expected interest rates. The r_t^n defined in (6.3) depends on the natural growth rate and the rate of temporal preference. If $\uparrow g_n \Rightarrow \uparrow u \Rightarrow \downarrow w \Rightarrow \downarrow v \Rightarrow \uparrow g_w$, therefore, a higher g_n leads to a higher r_n. However, in this model there is the possibility of involuntary unemployment due to the existence of some kind of price rigidities.

According to the slides, in equation (6.2): '*the output gap is different from zero when the rate of interest is not at its natural level*". However, by the fisher equation $r_t^n = i_t - E(\pi_{t+1}) = r_t$, being r_t the *real interest rate*, hence if $r_t = r_t^n \Rightarrow \sigma\{r_t - r_t^n\} = 0 \Rightarrow x_t = E(x_{t+1})$: wouldn't then the statement should be: "by equation (6.2), if at time t, the real interest rate, $r_t = i_t - E(\pi_{t+1})$, is equal to the natural interest rate, r_t^n, then the output gap, x_t, is equal to the expected output gap: $E(x_{t+1})$"? I just don't see how the output gap is zero.

Why is the NK model *keynesian*? Because they revive the policy prescriptions of the IS-LM models after the New Classical Macroeconomics: however, this policy prescriptions only have an effect on the short run, and, monetary factors do not affect the growth rate in the long run.

6.2 The Post-Keynesian Theories

These theories have some common characteristics: (I) the possibility of under employment equilibrium results stemming from a lack of effective demand; (II) the long-run extension of the principle of effective demand; (III) a monetary nature-explanation of the interest rate; (IV) money endogeneity; and an (V) insensitivity of aggregate demand to the interest rate.

With respect to (I), **Why is it that in this framework wage and prices flexibility does not ensure a tendency towards full employment?** (i) Let $c = \omega c_w + (1-\omega)c_r$ be the overall marginal propensity to consume, and assume that $c_w > c_r$, if $\downarrow \omega \Rightarrow \downarrow c \Rightarrow \downarrow Y \Rightarrow \downarrow L$. A fall in wages and prices can alter the wage share, and hence, also the marginal propensity to consume. (ii) If $\downarrow w \Rightarrow \downarrow P \Rightarrow \uparrow \frac{M}{P} \Rightarrow \downarrow i \Rightarrow$. **This may not happen if: money is endogenous and if the economy is near (or in) liquidity trap conditions.** (iii) Even if the rate of interest falls, investments may not increase: the curve of marginal efficiency of capital can be inelastic to the rate of interest (i.e., pessimistic expectations). Moreover, the fall in prices may induce bank failures (due to the disruptive effects that it has con debt-credit relationships), for an increase in the non-performing loans and credit rationing (Fisher's effect).

With respect to (II). Two routes to extend the principle of effective demand to the long-run. (a, the post-keynesian approach) if $S = s_w W + s_r R$, letting $s_w = 0$, we have that $g_K = \frac{I}{K} = \frac{S}{K} = \frac{s_r R}{K}(\frac{Y}{Y}) = \frac{s_r \pi}{v} = s_r r$. **An increase in the growth rate of capital accumulation implies an increase in r, taking the marginal propensity to save of profit earners (s_r) as given.** (b, the classical-keynesian approach) $\frac{S}{K}$ adjusts towards $\frac{I}{K}$ through changes in v and in K, without the need to change income distribution: **savings create additional productive capacity and in this way the actual degree of capacity utilization comes back to its normal or warranted value.** In this second route the trend of output is defined by the trend of the *autonomous components of aggregate demand*.

With respect to (V). **Consumption is influenced more by habits, current income, than by the interest rate.** Investment depends on expected changes in aggregate demand given the warranted capital-output ratio rather than on changes in the interest rate ($\Delta K =$

$I = v\Delta Y$). Assume and economy that produces a single commodity with two techniques (θ and γ) and two factors (a and b), one of which is capital. The relative price of the factors is such that $\frac{p_a}{p_b} \Rightarrow \frac{a_\theta}{b_\theta} > \frac{a_\gamma}{b_\gamma}$, hence γ is the cost minimizing technique: it will not be necessarily true that if $\downarrow \frac{p_a}{p_b} \Rightarrow \downarrow \frac{p_\theta}{p_\gamma}$ will fall: technique θ might no be adopted. This are some of the conclusions reach by the *Cambridge capital controversies*, in the 50's, 60's and 70's. Mainly, that we cannot order techniques according to their capital intensity independently of the rate of profit: relative prices are no independent of distribution (from this results it is that the phenomena of *reswitching* and *reverse capital deepening* appear). **This results imply that the neoclassical mechanism of direct and indirect substitution between factors of production may not work in under the assumption of heterogeneous capital goods.**

A further element explaining the insensitivity of investments to the rate of interest, if when it falls, prices fall, the expected rate falls, which produces no incentive to invest. **In this case the adjustment of the rate of profits to the interest rate does not pass through an increase in investments that determines a fall in the marginal efficiency of capital.** However there are possible effects of changes in the interest rate on aggregate demand: it can change the demand for durable consumption goods; it can have an influence on the investments of small firms; it can have an influence on export (through appreciations in the exchange rate); it may imply lower wages, hence, lower consumption; and it may also have an indirect effect on public expenditure due to changes in the cost of servicing the public debt.

With respect to (IV). **The condition of money endogeneity, plus the assumption of an insensitivity of investments regarding the rate of interest implies:** (i) no crowding out of private expenditure, rather, *crowding in*; (ii) fiscal policy is more effective than monetary policy in order to ensure full employment; (iii) rigid wages are a consequence rather than a cause of unemployment. From graphical point of view, the IS curve would be vertical or quasi-vertical (if not completely vertical), the LM curve would be horizontal: monetary policy is ineffective in a closed economy, with a quasi-vertical IS curve monetary policy would be effective, to a certain degree, however, fiscal policy would be completely effective and equal to the keynesian multiplier. In the AD-AS framework, the AS curve would be horizontal: $p = \frac{w}{q}(1+\mu)$, and the AD would also be quasi-vertical or vertical. **Hence: changes in prices do not affect aggregate demand, *fisher effects* bend the curve backwards: falls in money wages may lead to a fall in output.**

In post-keynesian theory there is also the possibility of *cost-push inflation*, with a price definition:

$$p = \frac{w}{q}(1+\mu) + ep^F Z(1+\mu) \tag{6.5}$$

According to equation (6.5) prices increase due to: increases in money wages (w), depreciation of the exchange rate (e), worsening in the terms of trade (the foreign imported prices p^F), increases in the profit margin (μ), and decreases in labor productivity (q). For a system of production with n commodities:

$$\mathbf{p} = (\mathbf{Ap} + \mathbf{Zp^F}e)\mathbf{\Omega} + w\mathbf{l} \tag{6.6}$$

Where $\mathbf{A}, \mathbf{Z}, \mathbf{\Omega} \in M_{n\times n}(\mathbb{R}^+)$ and $\mathbf{p}, \mathbf{p^F}, \mathbf{l} \in M_{1\times n}(\mathbb{R}^+)$. Moreover, note that we have have a mark-up: $\mathbf{\Omega} = diag(\omega_1, \omega_2, ..., \omega_n)$ is the diagonal matrix of profit terms $\omega_j = 1 + \rho_j = 1 + i + np_j$. Given the technique of production $(\mathbf{A}, \mathbf{Z}, \mathbf{l})$, the money wages ($w$), the exchange rate ($e$), the normal profit rate of enterprise (np_j), we have that $\uparrow i \Rightarrow \uparrow p \Rightarrow \downarrow w/p \wedge \uparrow r$. **We can define *cost-push inflation*:** taken i and ρ_{ij}, the price level increases when: money wages rise (w), the prices of imported inputs increase ($\mathbf{p^F}$), the mark-up on prices increases through increases in the interest rate (i) or the normal profit rate of enterprise (np_j).

This simple price model implies: (i) a direct explanation of the *gibson paradox* (the co-movement of prices and interest rates); (ii) a *distributive effect of monetary policy*: monetary institutions have an influence in income distribution when setting the nominal interest rate. (iii) **A conflict theory of distribution:** increases in money wages only have an indirect effect on distribution via increases in prices (which may provoke a reaction from the monetary authorities to increase the interest rates). A continuous increase in w implies a fall in the real interest rate as long as monetary authorities do not increase the nominal interest rate ($r = i - \pi^e$). **In this framework: income distribution is determined by the interactions between the decisions of monetary authorities and the trend of money wages.**

Suppose that the CB rises the nominal interest such that the real interest rate is r_{CB}, associated to this real interest rate there is wages w_{CB}: workers react asking for an increase in wages such $w_L > w_{CB}$. This wage can be defined as:

$$w_L = \varepsilon_0 + \varepsilon_1 u + \varepsilon_2 \tau + \varepsilon_3 y \qquad (6.7)$$

Where u is the unemployment rate, τ is a proxy of workers degree of organization (i.e., trade union membership), y is labor productivity and $\varepsilon_0 = a$ is a *measure of the influence of social and political factors.* CB's can change r_{CB} due to certain inflation target decisions, the effect of increasing inflation on real savings, fixed incomes and external constraints. Workers may reduce w_L, given unemployment, due to the negative effects that inflation has on the real wages of less organized workers, and fears regarding the implementation of restrictive fiscal and monetary policies. ***The average real wage*** **will be influenced by the** ***reaction functions*** **of workers, CB's and firms:** that is, the "speed" by which wage bargaining adjusts nominal wages, CB's change the nominal interest rate and firms transfer the increase in nominal interest rates onto prices.

Chapter 7

The Transmission Channels of Monetary Policies

Different theories convey different conclusions regarding the effects that different monetary policies have, and the transmission channels in which they operate, on activity levels. **The main channels identified by the literature are:** (a) monetary; (b) equity; (c) credit; (d) exchange; and (e) wealth. **(*a*, the monetary channel)** We have that $\downarrow M \Rightarrow \uparrow i \Rightarrow \downarrow I, C \Rightarrow \downarrow Y$: however this mechanism fails if aggregate demand is insensitive to the rate of interest.

(*b*, the equity channel) changes in i imply changes in the prices of bonds and equities, hence altering the amount of investments. According to Tobin, if $q = \frac{M}{R} = \frac{r_e}{r_m} > 1 \Rightarrow \uparrow I$. If the stock market value of a firm (M) is higher that the replacement cost of the firm's capital (R) then firms have an incentive to finance investments through the stock market. The same occurs if the internal rate of return on capital (r_e) is higher than the market interest rate (r_m): if $r_e > r_m \Rightarrow \uparrow I \Rightarrow \downarrow r_e \rightarrow r_m$. It also may occur via the *Pigou effect*: a rise in the private sector real net wealth induces an increase in consumption when stock prices increase.

(*c*, the credit channel) This channel has been of particular attention to new-keynesian authors, they've come up with a series of channels to explain this relationship. (i) bank lending channel: firms do not finance investments with the stock and bond markets, rather with their own funds or bank loans (due to asymmetric information and small dimensions). If the CB reduces the monetary base then there is fall in the amount of loans, which implies a decrease in investment and consumption. (*ii*) the balance sheet channel: (*i.1*) a fall in asset prices implies a lower value of collateral and hence a lower amount of loans; (*ii.2*) an increase in the interest rate implies a lower cash flow and hence a lower amount of loans. **(*d*, the exchange rate channel)** If $\downarrow M \Rightarrow \uparrow i \Rightarrow i > i^F \Rightarrow$ capital inflows $\Rightarrow \downarrow e \Rightarrow \downarrow X - M \Rightarrow \downarrow Y$.

(*e*, the wealth channel) Some preliminary elements. (*i*) Here there is a distinction between *real crowding out* (deletion of private expenditures due to bottlenecks): they might imply inflation; and *financial crowding out*: this is the usual indirect negative effect of deficit spending on private expenditure due to the *monetary retroactive effect*. (*ii*) The definition of outside money (activities that do not represent credit instruments on internal operators) and inside money (credit instruments on internal operators, i.e., private bonds, bank deposits, etc). An increase in outside money does not lead to an increase in the net wealth of the private sector only if it is achieved via open market operations: we can observe changes in the form of wealth (the monetary base-bonds relationship) but not in its size. In the case of deficit spending there is an increase in the net private wealth $G + iB = T + \Delta M + \Delta B$.

A different position is the following: public bonds are not part of W because they ought to be repaid by taxes. However: postponement of repayment can be extended to an infinite horizon; the interests on B can be paid by issuing new bonds and money; taxes for bond repayment are not paid by the present generation; public deficits generate private savings that otherwise would not materialize. Given $S - I = G - T$, only if $Y = Y_p \Rightarrow$ an increase in G implies a decrease in C or I.

An increase in outside money given B implies an increase in wealth: (*i*) we have the usual *real balance effect* (that doesn't work if the fall in $\frac{M}{P}$ is achieved through deflation). (*ii*) The effects on the money market in the presence of liquidity trap conditions, the assumption of money endogeneity and a change in the interest rate in the presence of an excess in money supply. (*iii*) The effect on the activities markets due to an increase in M are: (A) households buy patrimonial activities; (B) firms change their debt and credit positions. This has an effect on the goods market due to: (*1*) a revaluation effect (the change in the price of activities: direct effects on consumption and on the value of collaterals); an (*2*) availability effect (a possible increase in loans); and a (*3*) a yield effect, that can be separated into two distinct effects: the substitution effect (a shift towards activities with higher yields) and the income effect (that depends on the propensity to consume of debtors and creditors).

There is an alternative view regarding wealth effects: a wide range of financial activities are not perfectly substitutable with each other, hence, there is not a single rate if interest for transmission of monetary policy as in standard macroeconomic models. For Tobin: (i) there is a high substitutability between money and bonds an a low substitutability between financial and real activities; (ii) the stocks influence the flows.

Suppose that the government emits a certain amount of bonds. If M and B are highly substitutable and B and K are not (or vice-versa), then a low (strong) increase in the rate of interest will induce the private sector, the low (strong) increase in the rate of interest will have low (strong) effect on investments, $\frac{K}{W}$ falls, which implies a positive effect on the amount of investments in order to attain the desired proportion of K in the overall wealth, however the wealth effect $>$ the substitution effect (the wealth effect $<$ the substitution effect) the demand for capital rises (falls). For Tobin: the direction of changes in M and i are not the only factor that needs to be taken into account in order to understand what happens to investments.

7.1 Uncertainty, Rules and Discretion

As seen previously, the implementation of monetary policies have to take into account the complexity of portfolio adjustments, however, the also need to take into account the uncertainty of economic conditions and the lags in the effects of monetary policies. In conditions of certainty: M and i are indifferent instruments of monetary policy; one instrument for each objective (*Tinbergen criterion*). Conditions of uncertainty influence the appropriate instrument of monetary policy, the existence of such conditions has been used as to argue that CB's shouldn't adopt fixed rules.

There are different types of uncertainty: (i) on the value of the parameters (structural); (ii) due to information lags; (iii) regarding the origin of disturbances; and (iv) on the dynamical relations between variables and the lag structure (dynamical). **The *Poole model* is a model regarding the appropriate choice of monetary policy instruments in conditions of uncertainty.** The monetary authorities want to stabilize Y around a certain value Y^*. If there is uncertainty in the goods market (money market), monetary authorities then should fix M (i) and permit that i (M) varies between a certain range.

If real disturbances are stronger than the monetary ones, then the lower is the sensitivity of the demand for money to rate of interest: the more vertical the LM curve is and the lower are output fluctuations around Y^*. For the monetarists: the financial system needs to be reformed in order to obtain a closer connection between money and income. This last position was the prevailing one during the 1980's (rules against discretion). For Poole: discretionary policies should no be excluded, the model is limited by the assumption that CB's control M. **The classification of monetary regimes:** (i) passive simple rules (*Friedman's rule*, pegged exchange rates); (ii) active simple rules: intermediate monetary and non-monetary objectives with active rules, plus, a final objective with adjustments of the instruments (inflation targeting).

7.2 Inflation Targeting

This regime implies a lower influence on the decisions of monetary authorities in relationship to other objectives (such a full employment), in real life there has been a higher or lower mechanical application of the rule. **The model can be summarized by the following system of equations:**

$$x_t = f(x_{t-1}, i_t - \pi_t^e, E_t) + u_t \text{ (dynamic IS curve)} \tag{7.1}$$

$$\pi_t = g(x_t, \pi_{t-1}, E_t) + v_t \text{ (Phillips curve)} \tag{7.2}$$

$$E_t = h(E_{t-1}) + \varepsilon_t \text{ (trend in exogenous variables)} \tag{7.3}$$

$$L = \frac{1}{2}[x_t^2 + \beta(\pi_t - \pi^T)^2] \text{ (loss function of the CB)} \tag{7.4}$$

$$\Delta i_t = \alpha(\pi_{t+j}^e - \pi^T) \text{ (the reaction function)} \tag{7.5}$$

Where, at time t, x_t is the output gap, i_t is the nominal interest rate, π_t^e is the expected inflation rate, π^T is the targeted inflation rate, E_t is a set of exogenous variables that are not under the control of monetary authorities, and $u, v, \varepsilon \sim \mathcal{N}(0, \sigma^2)$. The lost function can be minimized such that if $\beta = 1$ the CB has a balanced aversion between inflation and unemployment, **if $\beta < 1$ the CB is *unemployment averse* and if $\beta > 1$ the CB is**

inflation averse. The value j in π^e_{t+j} is derived from the lag structure: the CB knows that in the changes in i_t will have an effect on x_t through (7.1) and on π_t, after a certain number of periods through equation (7.2). The CB changes i_t in order to ensure that the inflation rate is equal to the expected inflation rate at tie $t+j$.

The inflation targeting regime is *forward looking*, monetary policy is oriented towards the future: the idea is to achieve the targeted inflation rate on average. The effectiveness of the regime is positively correlated to the CB's credibility: that is, the Cb influences the formation of expectations. **Some initial problems with inflation targeting**: (i) the CB should know the lag structure (if not, the CB may increase, rather than decrease, the variability of π_t when changing the rate of interest); (ii) how is π^T chosen?; (iii) it is assumed that $Y \to Y_p$: price flexibility sharpens output fluctuations, if we don't have that $Y \to Y_p$ then an inflation targeting regime may be an obstacle to full employment; (iv) it's suboptimal in the presence of supply shocks; (v) deviations of π_t from π^T might come from a distorted use of policy instruments or random factors: it is difficult to distinguish between this to causes.

The main characteristics of the inflation targeting models: (i) $Y \to Y_p$; (ii) the Phillips curve is vertical in the long-run, but not in short-run (due nominal wage and price rigidities); (iii) CB's have a reaction rule. **The model can be showcased in a simplified form:**

$$\pi_t = \pi_{t-1} + \alpha x = \pi_{t-1} + \alpha \frac{y - y_p}{y_p} \text{ (Phillips curve)} \tag{7.6}$$

$$y = y_0 - br \text{ (IS curve)} \tag{7.7}$$

$$\pi_t = \pi^T - ax \text{ (monetary policy rule)} \tag{7.8}$$

We can divide both sides of equation (7.7) by y_p, and if $y_p = y$, we have that $r_n = \frac{1}{b_1}\left(\frac{y_0}{y_p} - 1\right) = \frac{x_o}{b_1}$. With this new definition we can see that $x = x_o - b_1 r \Rightarrow x = b_1 r_n - b_1 r = b_1(r_n - r)$. Hence if $r_n = r \Rightarrow x = 0$, if $r_n > r \Rightarrow x > 0$ and if $r_n < r \Rightarrow x < 0$. By equalizing equation (7.6) and equation (7.8) we get:

$$\Delta r = h(\pi_{t-1} - \pi^T) \text{ where } h = \frac{1}{b_1(a + \alpha)} \tag{7.9}$$

This last equation is the reaction function of the CB. Some dynamics, from equilibrium $\pi_{t-1} = \pi^T = \pi^e$ If $\uparrow y \Rightarrow x > 0 \Rightarrow \pi_t > \pi^e(\uparrow r) \Rightarrow$ the Phillips Curve shifts upwards $\Rightarrow \pi_t < \pi^e(\downarrow r)$ until $x = 0$. **This new equilibrium is characterized by a higher natural interest rate.** Suppose that $\downarrow \pi^T$, then equation (7.8) shifts to downwards where $\pi_t > \pi^T \Rightarrow$ the CB rises its interest rate, here we have that $\pi_t > \pi^e \Rightarrow$ the Phillips curve shifts downwards. The CB lowers the rate of interest and output goes back to it's potential level.

7.3 The Taylor Rule

In the inflation targeting it is assumed that $Y \rightarrow Y_p$, through the usual neoclassical substitution mechanisms in consumption and production. The same applies when considering the most widespread reaction function:

$$i_t = r_n + \pi^* + \alpha(\pi_t - \pi^*) + \beta(y_t - y_p) \tag{7.10}$$

Where π^* is the expected inflation rate that is also equal to the targeted inflation rate, inflation targeting is a particular specification of the Taylor which modifies the lag structure and the way in which expectations are formed. The Taylor rule has been viewed as a benchmark for monetary policy and has become the pillar of New-Keynesian Dynamic Stochastic General Equilibrium (DSGE) models. Following equation (7.10), ***the Taylor principle*** says that the value of α should be such that as to ensure that if $\pi > \pi^* \Rightarrow \uparrow i \Rightarrow \uparrow r \Rightarrow r > r_n \Rightarrow \downarrow y \Rightarrow \downarrow \pi$.

There are various formulations of the Taylor rule:

$$i_t = r_n + \pi_t + \alpha(\pi_t - \pi^T) + \beta(y_t - y_p) \text{ (original, 1993)} \tag{7.11}$$

$$i_t = r_n + \pi_t^{core} + \alpha(\pi_t^{core} - \pi^T) + \beta(y_t - y_p) \text{ (core, 1999)} \tag{7.12}$$

$$i_t = \varrho i_{t-1} + (1-\varrho)[r_n + \pi_t^{core} + \alpha(\pi_t^{core} - \pi^T) + \beta(y_t - y_p)] \text{ (inertial, 1999)} \tag{7.13}$$

$$i_t = r_n + \pi_{t+j}^{core} + \alpha(\pi_{t+j}^{core} - \pi^T) + \beta(y_t - y_p) \text{ (forward looking, 2000's)} \tag{7.14}$$

$$i_t = \varrho i_{t-1} + (1-\varrho)[r_n + \pi_{t+j}^{core} + \alpha(\pi_{t+j}^{core} - \pi^T) + \beta(y_t - y_p)] \text{ (forward+inertial)} \tag{7.15}$$

The reflection of the Taylor rule as the actual behavior of CB's is sensitive to different hypothesis on the parameters and the measures of price inflation.

The rule might be found to fit by adjusting the parameters of the model and the values of unobservable variables (such as r_n and y_p). Also, it should be noted that the policy implications varies significantly depending on the particular chosen specification. **Some problems regarding the implementation of the Taylor rule by CB's:** (i) what notion should be considered for r_n? natural or *quasi-natural* rates have varying implications; (ii) the variety of methods that are used to estimate r_n: different methods lead to different interpretation regarding monetary policy (expansionary or restrictive?).

There, however, more relevant problems regarding the New-Keynesian approach. It is assumed that: (i) monetary policy has no effects in the long-run income distribution and output trends; (ii) increases in the rate of interest do not necessarily imply a fall in aggregate demand. **Some theoretical limits of the New-Keynesian approach**: (i) there might be a low and asymmetric elasticity of aggregate demand to the rate of interest; (ii) there might be a direct relationship between prices and the interest rate (the *gibson paradox*); and (iii) the existence in itself of a monetary-independent natural interest rate determined by *productivity and thrift* via the adjustment of the real interest rate towards a value assuring full employment, where, profit expectation govern the demand for credit.

Keynes, contrary to this latter point, argues that savings equalize investments by means of income changes: he viewed, in turn, the rate of interest as a monetary phenomenon to which capital profitability will adjust. **Keynes's criticisms are reinforced by the *Cambridge Capital Controversies* debate of the 1960's: it is impossible to derive a decreasing supply curve of firm investments with regard to the interest rate.** The results from this debates also put into question the assumed tendency of output towards potential output via *market forces*, assumption that underestimates the fact that output fluctuations may influence the actual trend of output.

7.4 The Gibson Paradox

According to the *Gibson Paradox* (also known as *price puzzle*), there can be a direct relationship between the level of prices and the interest rate: that is, a co-movement between long-term interest rates and prices. Some explanations. **According**

to Tooke: the interest rate is a component of the normal cost of production, increases in the interest rate lead to increases in the monetary costs of production and hence in commodity prices. For Wicksell, criticizing Tooke, in an economy with a money-comoodity, increases in the rate of interest induce a fall in the real wages: hence, the price level depends on the technical conditions of production of the money-commodity relative to the technical conditions of production of the other commodities.

According to Wicksell: the paradox is a result of the adjustment of the nominal interest rate towards the natural rate, such that if $r_n > i \Rightarrow$ *cumulative phase of inflation*, and, eventually, $i \rightarrow r_n$, during the Gibson paradox is observable. **According to Fisher:** the phenomenon is explained by the existence of *money illusion*, and the fact that changes in inflation rate due to higher growth rates in the money supply are not perfectly embodied in the nominal interest rates. Discrepancies between the real interest rate and the natural interest lead to fluctuations in output and prices around their trend levels (during the adjustment process it is that the Gibson paradox is observable).

A**ccording to a series of authors, this phenomenon is only observed, historically, in countries with a gold standard system.** Why? Increases in the rate of interest raise the cost of non-monetary gold changing positively the supply of monetary gold, while the demand of monetary gold decreases (if the demand for money is more elastic to the interest rate). Hence, there is a decreases in the demand of monetary gold in relationship to its supply and thus the prices of gold fall (rising the general level of prices).

However, the phenomenon has also been studied in the case of a *fiat money economy*, particularly in the 1990's: here is has been named *price puzzle*. In more recent literature, there are two main interpretations-explanations regarding this phenomenon: (i) it is explained due to identifications problems in the estimates of prices and interest rates; (ii) it is explained through a cost channel (prices rise when interest rates increases due to higher production costs).

Chapter 8

Money and Public Finance

In order to obtain the *intertemporal public budget constraint* we first have to define the budgets of the CB and the *treasury*:

$$G_t + i_{t-1}B^T_{t-1} = T_t + \Delta B^T_t + RCB_t \tag{8.1}$$

$$\Delta B^M_t + RCB_t = i_{t-1}B^M_{t-1} + \Delta H_t \tag{8.2}$$

For equation (8.1), we have that G_t is public expenditure, $i_{t-1}B^T_{t-1}$ is the amount interest paid on the total stock of public debt, and that this two expenses can be financed by: T_t (public taxes), ΔB^T_t (the issuing of new bonds), and RCB_t (the interests paid on the amount of public bonds paid by the CB to the treasury). For equation (8.2) we have that, for the CB, the change in the amount of public bonds held by the CB ΔB^M_t, plus RCB_t, most be equal (can be financed) to the interests received by the treasury for the bonds that the CB posses $i_{t-1}B^M_{t-1}$, plus the emission of monetary base (or high-powered money) ΔH_t. Let $B = B^T - B^M$, then we have that the budget of the public sector is:

$$G_t + i_{t-1}B_{t-1} = T_t + \Delta B_t + \Delta H_t \tag{8.3}$$

Dividing both sides of the equation by P_t and multiplying the terms associated to the last period by $\frac{P_{t-1}}{P_{t-1}}$, remembering that $1 + \pi_t = \frac{P_t}{P_{t-1}}$ and that $1 + r = \frac{1+i}{1+\pi}$ we obtain the public budget in real terms:

$$g_t + r_{t-1}b_{t-1} = t_t + \Delta b_t + h_t - \frac{h_{t-1}}{(1+\pi_t)} = \tag{8.4}$$

$$t_t + \Delta b_t + s_t = t_t + \Delta b_t + \Delta h_t + \underbrace{\frac{\pi_t}{(1+\pi_t)}h_{t-1}}_{\text{inflation tax}} \tag{8.5}$$

Here, it is important to note that r_{t-1} is the *ex post* real rate of interest, thus far no distinction has been made between anticipated and unanticipated inflation rates. The term s_t captures the *seigniorage*, that is, the power of the state to finance his expenses via his ability to coin money. **Assuming that r is constant over, time, we get that:**

$$\begin{aligned}
b_{t-1} + \frac{g_t}{(1+r)} &= \frac{t_t}{(1+r)} + \frac{s_t}{(1+r)} + \frac{b_t}{(1+r)} \\
b_t + \frac{g_{t+1}}{(1+r)} &= \frac{t_{t+1}}{(1+r)} + \frac{s_{t+1}}{(1+r)} + \frac{b_{t+1}}{(1+r)} \\
\Rightarrow b_{t-1} + \left(\frac{g_t}{1+r} + \frac{g_{t+1}}{(1+r)^2}\right) &= \left(\frac{t_t}{1+r} + \frac{t_{t+1}}{(1+r)^2}\right) \\
&\quad + \left(\frac{s_t}{1+r} + \frac{s_{t+1}}{(1+r)^2}\right) + \frac{b_{t+1}}{(1+r)^2}
\end{aligned}$$

$$\text{For } t \to \infty \Rightarrow (1+r)b_{t-1} + \sum_{i=0}^{\infty} \frac{g_{t+i}}{(1+r)^i} = \sum_{i=0}^{\infty} \frac{t_{t+i}}{(1+r)^i} + \sum_{i=0}^{\infty} \frac{s_{t+i}}{(1+r)^i} \tag{8.6}$$

Equation (8.6) is the intertemporal budget constraint of the public sector, assuming that $\lim_{i\to\infty} \frac{b_{t+i}}{(1+r)^i} = 0$ (no-ponzi fame condition).[1] Hence, the government must plan to obtain revenues by current and future taxes, and by current and future seigniorage. **The actual value of taxes and seigniorage must be equal to the current and future expenses of the public sector and to the amount needed to repay the initial public debt.** If $b_t > 0$, then the government must plan to have primary surpluses or the appropriate values of s.

From this framework we can identify two *dominance regimes*: (i) fiscal: the treasury fixes g and t over time, the CB fulfills the intertemporal public budget constraint through changes in s; and (ii) monetary: the CB fixes s (or the interest rate) over time, and the treasury adjusts t and g in order to comply with the intertemporal constraint. It is argued that there is a limit to the revenue that the state can obtain through seigniorage,

[1] This means that the state cannot postpone its debts indefinitely.

due to the *inflation tax*. Assume that public deficits can only be financed by issuing money, the uniperiodal public deficit is $d = g + ib - t$, and $s = \vartheta h$, where ϑ is the growth rate of the nominal monetary base H. Note that $\ln(h) = \ln(H) - \ln(P) \Rightarrow \vartheta_r = \vartheta - \pi \Rightarrow d = s = (\vartheta_r + \pi)h = \Delta h + h\pi$. **Which, just as in equation (8.5), means that s is determined by the real change in money supply and an inflation tax**. If (i) r is determined by real factors; (ii) $\vartheta = \pi$; (iii) the desired amount of real balances is a decreasing function of $\pi \Rightarrow \exists \pi_{max}$ for which seigniorage reaches its maximum value (s_{max}). If $\pi < \pi_{max} \Rightarrow \uparrow s$, and vice-versa. **If $d > s_{max}$ then it is impossible to establish a fiscal dominance regime, under the specific conditions mentioned above.**

When discarding seigniorage primary surpluses are needed to repay the initial public debt, however, over time you could have public deficits financed by issuing bonds. The engulfment of the no-ponzi conditions is not incompatible with an increase in public debt over time. Let γ be the growth rate of public debt, and let $r > \gamma$, then

$$\lim_{t\to\infty} \frac{B_t}{(1+r)^t} = B_0 \lim_{t\to\infty} \left(\frac{1+\gamma}{1+r}\right)^t = 0 \tag{8.7}$$

The limit is zero as long as the real public debt rises at a rate that is lower than the interest rate. In order to discuss the sustainability of public debt is necessary introduce a definite time horizon. **Why does the public debt must be repaid or kept under control?** (i) the extension to the state, of the set of conditions that only hold for the private sector; (ii) the notion that $\uparrow \frac{B}{Y} \Rightarrow \uparrow i$, thus becoming unsustainable over time; and (iii) the idea that d determines a lower amount of private expenditure: public debt substitutes other forms of wealth, reducing capital accumulation, labor productivity and the growth per capital consumption.

Some additional discussion elements regarding the sustainability of public debt: (i) $\frac{B}{Y}$ compares a stock B with a flow Y, a different picture arises from comparing the debt-to-wealth ratio; (ii) it is considered that public debt is sustainable as long as $\frac{B}{Y}$ remains constant over time; and (iii) a constant $\frac{B}{Y}$ does not require public surpluses. If $G = T$ and the service of public debt is paid by issuing bonds then $\frac{B}{Y}$ remains unchanged as long as $g = i$, and it decreases (increases) if $i < g$ $(i > g)$. Moreover, it is also possible to have a decreasing $\frac{B}{Y}$ even in the presence of primary deficits. Assuming that $\Delta H = 0$, let $\phi = \frac{G-T}{B}$

and let $g = \frac{\Delta Y}{Y}$, we have that:

$$G + iB = T + \Delta B$$
$$\Delta B = G - T + iB$$
$$\gamma = \phi + i$$
$$\Rightarrow \text{If } g > \phi + i \Rightarrow \downarrow \frac{B}{Y}$$

This last inequality implies that, as long as the growth rate of the economy (g) is higher than the growth rate of the public debt ($\phi + i$), the public debt to income ratio ($\frac{B}{Y}$) will fall. (iv) Regarding the crowding out of private investment, in a closed economy where transfers by the state are identified only with the interests R_t paid on the stock of public debt then, the savings of the private sector are:

$$S_p = (1-c)(1-t)Y_t + (1-c_r)R_t$$
$$\Rightarrow (1+c_r)R_t + (1-c)(1-t)Y_t - I_t = G_t + R_t - T_t = D$$

It is only when $Y_t = Y_p$ that increases in G lead to decreases in I. If $Y \neq Y_p$ then public deficits bring about savings that otherwise would not occur. D is added to I in determining savings, its changes alter the net wealth of the private sector. (v) According to Barro B is not a part of the net wealth of the private sector: if d is financed by issuing bonds, then there is a fall in private consumption in order to pay in the next period the amount of taxes need to repay the public debt. However, public deficits generate savings without any need of a change in private consumption. (vi) A long-run extension of the principle of effective demand would imply that attempts at reducing $\frac{B}{Y}$ via primary surpluses may lead to a rise of such ratio. We have that, if $(1-c)(1-t) < \frac{B_a}{Y_a} \Rightarrow \downarrow G \Rightarrow \downarrow D \Rightarrow \uparrow \frac{B}{Y}$, where $\frac{B_a}{Y_a}$ is the debt-income ratio without the implementation of a restrictive fiscal policy. Let $m = \frac{1}{1-c(1-t)}$, the debt-income ratio with the implementation of a restrictive fiscal policy is:

$$\frac{B_r}{Y_r} = \frac{B_a - \Delta D}{Y_a - m\Delta G} = \frac{B_a - \Delta G(1-tm)}{Y_a - m\Delta G} \tag{8.8}$$

This implies that $\frac{Br}{Br} < \frac{B_a}{Y_a}$ if $\frac{B_a - \Delta G(1-tm)}{Y_a - m\Delta G} < \frac{B_a}{Y_a} \Rightarrow \frac{1}{m} - t > \frac{B_a}{Y_a}$. However, **in specific circumstances public debt can be a problem**: (i) open economy (falls in the demand for bonds in balance of payment crisis situations); (*ii*) bounded monetary sovereignty (excessive power of the rentiers); (*iii*) portfolio effects (changes in the preference for liquidity); and (*iv*) distributive issues (the amount of interests paid to rentiers with a high public debt).

Chapter 9

Some Monetary Issues in an Open Economy

9.1 Conflict Inflation in an Open Economy

Changes in the real exchange rate have distributive effects. Consider the price system:

$$\mathbf{pB} = (1+\rho)(\mathbf{pA} + e\mathbf{p^F M}) + w\mathbf{l} \tag{9.1}$$

Where $\mathbf{B}$ is the diagonal matrix of outputs, $\mathbf{A}$ is the matrix of technical coefficients, $\mathbf{M}$ is the matrix of imported inputs, e is the nominal exchange rates, $\mathbf{p^F}$ is the price vector of imported goods, w is the nominal wage rate and $\mathbf{l}$ is the labor vector. Let $\mathbf{p}(\mathbf{B}-\mathbf{A}) = \mathbf{pY}$, where $\mathbf{Y}$ is the vector of domestic inputs, then, dividing both sides of the equation by it we get:

$$\frac{w_r}{y} + \varepsilon_r m = 1 - q_p \tag{9.2}$$

Where w_r is the real wage, y is labor productivity, ε_r is the real exchange rate, m is the weight of imports in the value added and q_p is the share of profits. Then, we will have than an appreciation in the exchange rate will lead to a rise in the real wage, surely, when such appreciation is achieved without rising interest rates. If an appreciation is achieved via rises in the interest rate, then the final effect on the real wage will depend on many different circumstances (the weight of imported goods in the production process, in the bundle of consumption of workers, etc). In case of a depreciation, the outcome depends on the bargaining power of workers.

9.2 The Limits of the Mundell-Fleming Model

In this model, in addition to the usual IS-LM curves, a BP curve is introduced in order to take into account the conditions of balance-of-payments equilibrium, which depends on the conditions of the trade account and financial account. The model can be written in the following way:

$$Y = \frac{1}{1 - c_1(1 - t_1) + z}(c_0 + I_0 + X_0 - Z_0) - bi = m(A - bi) \text{ (IS)} \tag{9.3}$$

$$i = \frac{1}{l_1}[kY + (l_0 - \bar{M}_r)] \text{ (LM)} \tag{9.4}$$

$$i = i_F + \frac{1}{\varphi}(Z_0 - zY - X_0) \text{ (BP)} \tag{9.5}$$

Where z is the propensity to import, Z_0 is an autonomous component of imports and i_F is the foreign interest rate. Equation (9.5) shows all the pairs (i, Y) such that $BP = (X - Z) + \varphi(i - i_F) = 0$. Where φ is a sensitivity measure of capital movements with respect to interest rate differential. **It must be noted that if e is fixed, then M_s is endogenous.** Also, note that $\left|\frac{\Delta X/X}{\Delta e/e} + \frac{\Delta M/M}{\Delta e/e}\right| > 1$ (Marshall-Lerner condition).

Under the assumption of perfect capital mobility. We have the following two cases, regarding the exchange rate regime. (i) if e is not fixed $\Rightarrow$ expansionary monetary policies are effective: $\uparrow M_s \Rightarrow \downarrow i \Rightarrow \uparrow I \wedge \uparrow e \Rightarrow \uparrow X - Z_0 \Rightarrow$ IS shifts to the right. In this context expansionary fiscal policies increase the interest rate inducing private investment crowding out and an appreciation of the exchange rate (net export contraction). (ii) if e is fixed $\Rightarrow$ expansionary fiscal policies are effective: $\uparrow G \Rightarrow \uparrow Y \Rightarrow \uparrow i \Rightarrow \downarrow e \Rightarrow \uparrow M_s \Rightarrow \downarrow i \Rightarrow$ the IS shifts to the right. In this case, shifts in the LM curve return to their previous positions, making monetary policy ineffective as a tool to increase output.

If the BP curve slopes upward, then, the international interest rate increases when the level of output increases: higher output brings with it higher imports. Why? Agents might require higher premiums to retain the same assets in larger quantities, market imperfections might increase the loan risks when a country increases its liabilities denominated in foreign currency, etc. We identify two particular cases: (a) if $\frac{\Delta i_F/i_F}{\Delta Y/Y} < \frac{\Delta i/i}{\Delta Y/Y}$ then there is *high capital mobility*; and (b) if $\frac{\Delta i_F/i_F}{\Delta Y/Y} > \frac{\Delta i/i}{\Delta Y/Y}$ then there is *low capital mobility.* In a flexible exchange rate regime, taking into account that the IS and BP curves shift together (for changes in the real exchange rate), we have that, for expansionary fiscal policies: if (a) $\Rightarrow\uparrow G \Rightarrow\uparrow Y \Rightarrow\uparrow i \Rightarrow\downarrow e \Rightarrow\downarrow X - Z \Rightarrow$ BP shifts to the left $\Rightarrow\uparrow i_F$, however, given that $\frac{\Delta i_F/i_F}{\Delta Y/Y} < \frac{\Delta i/i}{\Delta Y/Y} \Rightarrow i_F < i \Rightarrow\downarrow e \Rightarrow\downarrow Y(\downarrow X - Z) \Rightarrow\downarrow i \rightarrow i_F$; if ($b$) $\Rightarrow\uparrow G \Rightarrow\uparrow Y \Rightarrow\uparrow i$, however, since $\frac{\Delta i_F/i_F}{\Delta Y/Y} > \frac{\Delta i/i}{\Delta Y/Y} \Rightarrow\uparrow e \Rightarrow\uparrow X - Z \Rightarrow$ BP shifts to right: this process stops when $i = i_F$. In real life: the LM curve is always horizontal, its slope is irrelevant, as in practice CB's set the domestic interest rates.

A fixed exchange rate with a horizontal LM curve (endogenous money). Some issues: (i) the idea of an unlimited supply of foreign capital as long as $i > i_F$ (beyond a certain limit foreign credit rationing is bound to happen); (ii) as an economy grows, we have that $\uparrow Y \Rightarrow\uparrow i \Rightarrow\downarrow e \Rightarrow\downarrow X - M \Rightarrow$ decrease in international reserves: there may be growing doubts about the sustainability of the fixed exchange rate (there might no be *full credibility of the peg*).

In regards to the effectiveness of monetary policy in a fixed exchange rate regime, if the CB sets the nominal interest rate, then the mechanisms of the Mundell-Fleming model make no sense: (i) capital inflows might leave the monetary base unchanged (the inflow of capital if offset directly by an equal increase in the domestic public debt, i.e., all capital is used to buy bonds); (ii) different institutional arrangements reach the same results as money endogeneity; (iii) increases in reserves do not necessarily need to alter the domestic interest rates, there is no reason to suppose that the banks are forced to lend these additional resources if the number of costumers remains the same.

In a context of a fixed exchange rate regime with free capital mobility there is two asymmetries. (i) Increasing or decreasing international reserves (i is above or below i_F). For $i > i_F$, we have that technically there is no limit to accumulation, increasing reserves $\Rightarrow$ increasing expansion of the public debt due to sterilization processes (hence increasing transfers of income to public debt holders). However such transfers do not have an impact on aggregate demand. In turn, when in comes to decreasing international reserves ($i < i_F$): reserves are finite and can fall very rapidly in the light of expectations of an eventual devaluation of the exchange rate. The latter movements can quickly make a fixed exchange regime impossible, when $i < i_F$ the BP curve shifts upwards because there is an increase in the external spread due to the loss of reserves, plus, as mentioned before, expectations regarding a devaluation of the exchange rate increase.

(ii) The other important asymmetry in fixed exchange rate regimes with free capital mobility is between expansionary and contractionary policies. If $i > i_F \Rightarrow$ expansionary monetary or fiscal policies lead to an increase in Z and a loss of international reserves: worsening external debt sustainability and the economy's external liquidity. On the other hand, contractionary monetary or fiscal policies (independently of the position of i with respect to i_F) have the opposite effect: they reduce the current-account deficit and increase international reserves.

The case of floating exchange rates. For an exogenous nominal interest rate, if $i < i_F \Rightarrow \uparrow e$, even for an expanding economy, i will not rise: M_s moves in line with the economy's increasing levels of activity. However, if $i = i_F$, then expansionary fiscal policies will not be offset by the fall in net exports provoked by an appreciation of the exchange rate. Also, note that in the Mundell-Fleming model: $\uparrow e \Rightarrow \downarrow w_r \Rightarrow \downarrow c \Rightarrow \downarrow C \Rightarrow \downarrow Y$. **The latter point is counterexample of one the main conclusions of the Mundell-Fleming model: exchange rate deprecations are always expansionary.**

Also, if the country's private companies and/or banks are highly indebted in foreign currency, exchange rate depreciation might induce financial crisis. Also, abandoning the hypothesis of rigid expectations regarding the exchange rate, such that these expectations are elastic with respect to the actually observed exchange rate, then, the Mundell-Fleming model becomes quite unstable: there are continuous shifts in the BP curve. Hence, in real life, in a floating exchange rate regime the CB intervenes in the foreign exchange market and varying the domestic interest rate in order to control de direction and variability of the nominal exchange rate.

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