Theories of Distribution, Employment and Growth

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Contents

Chapter 1

The Analytical Structure of Neoclassical Distribution Theory

1.1 Preamble

These lecture notes were taken during the winter semester (2025-2026) at the Economics Department of Roma Tre University, of the *Theories of Distribution, Employment and Growth* course, adapted from the slides (and class notes) of professor Antonella Stirati. We also consulted and supported this notes with Petri (2021).

1.2 Structure and Assumptions

A *theory of distribution* is the part of economic theory that explains how, and by which forces, the product (or income) of a society is divided between various income categories, usually being: wages, profits and rents. In the neoclassical theory of distribution we can identify the following characteristics: (*i*) wages and profits are both determined via supply and demand in the markets for labor and capital, respectively; (*ii*) the determination of wages and profits is determined symmetrically and simultaneously by the same mechanisms; (*iii*) there is no space for any notion of *surplus* or residual.

In this sense is that we can affirm that the neoclassical theory of distribution is "subsumed" within the neoclassical theory of value: in the sense that the same forces that determine the prices of commodities, supply and demand, determine factor rewards. Furthermore, we can say that its production theory is conceived as the cooperation of factors of production under a specific combination, or proportion that remains fixed during the production process (a technique of production). Variations in the proportions of this factors (while producing the same amount of goods) implies switching between production techniques. **In a one good economy** (where that good is also used as capital), we can proceed to enunciate the *givens*, or *data*, that the neoclassical theory uses for its theory of distribution:

1. Consumer preferences. (i) We know that $\exists X = \mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$, where X is the consumption set with a preference relationship $\succeq \subseteq X \times X$ so that $\forall x, y, z \in X$ then: $\succeq y \vee y \succeq x$ (completeness); (ii) $x \succeq y \wedge y \succeq z \Rightarrow x \succeq z$ (transitivity); (iii) if $x > y \Rightarrow x \succ y$ (monotone preference relation); and (iv) for any sequence of pairs $\{(x^n, y^n)\}_{n=1}^{\infty}$ where $x^n \succeq y^n$ $\forall n \in \mathbb{N}$, $x = \lim_{n\to\infty} x^n \wedge y = \lim_{n\to\infty} y^n \Rightarrow x \succeq y$. Furthermore, we can define a utility to function $u : X : \mathbb{R}_+ \to \mathbb{R}$ to represent $\succeq \subseteq X \times X$ if $\forall x, y$ we have that $x \succeq y \iff u(x) \geq u(y)$, where $u'(x) > 0 \wedge u''(x) < 0$.

2. The technical conditions of production. We have a production function $f : \mathbb{R}_+^2 \to \mathbb{R}^+$, $Y = f(K, L)$ that satisfies: (i) $tY = tf(K, L) = f(tK, tL)$ $\forall t > 0$ (constant returns to scale); $MP_L = \frac{\partial Y}{\partial L} > 0 \wedge MP_K = \frac{\partial Y}{\partial K} > 0$, $\frac{\partial^2 Y}{\partial L^2} < 0 \wedge \frac{\partial^2 Y}{\partial K^2} < 0$ (decreasing marginal productivity for L and K); and (iii) the *Inada Conditions*.

3. Factor endowments. The total quantities of L and K in the economy expressed as scalars $\bar{L} > 0$ and $\bar{K} > 0$.

1.3 Monotonically Decreasing Demand Functions for Labor and Capital

1.3.1 Direct Substitution, in Production

Profit maximization behavior in firms, being price takers, implies that:

$$(i) \text{ For a fixed } \bar{K} \max_L (y - wL) \Rightarrow \frac{\partial y}{\partial L} - w = 0 \Rightarrow w = \frac{\partial y}{\partial L};$$

$$(ii)\text{ For a fixed } \bar{L} \max_{K}(y - rK) \Rightarrow \frac{\partial y}{\partial K} - r = 0 \Rightarrow r = \frac{\partial y}{\partial K}$$

Assuming a fixed endowment of capital, and that production is organized by capitalists, we can map a curve in the $(L, \frac{\partial Y}{\partial L})$ space where we express MP_L as a function of the amount of labor employed by the firm (see Figure 1.1). The horizontal segment of the curve exists because MP_K becomes negative when $k = \frac{K}{L}$ becomes too large (L is close to zero). For very small levels of L it is better for firms to only use the amount of capital that allows $MP_K = 0$. As we can observe in Figure 1.1, if the firm decides to demand $L_1 \Rightarrow w_1 > w^* = MP_L$ (the firm would be paying above MP_L, and thus losing space for possible profits), and if they demand $L_2 \Rightarrow w_2 < w^* = MP_L$ laborers wouldn't want to join a firm that pays below the market average w^*.

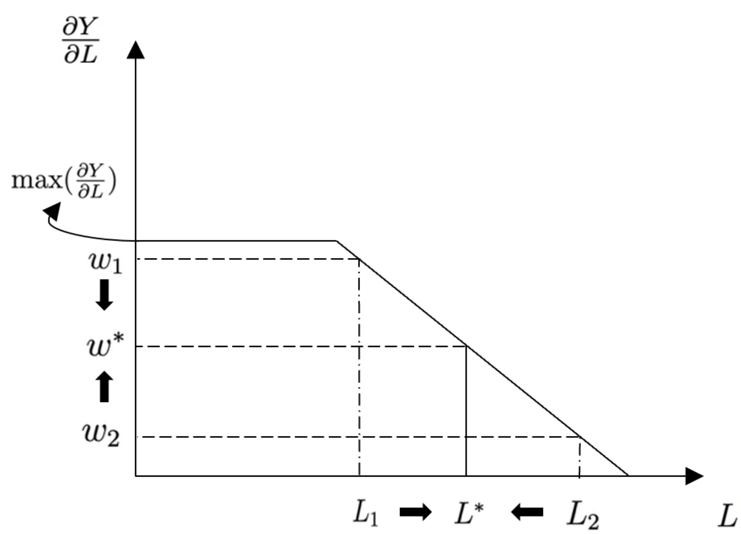

Figure 1.1: Marginal product of labor as the firms demand curve for labor.

Notice how since all firms are price takers of $w^* = MP_L$, all will tend to demand the amount of labor that reaches this equality: which could be translated as to saying that all firms adopt the same capital-to-labor ratio k (the same technique of production). We can define $L/K = \mu$, let μ^* be L/K ratio such that $MP_L = w^*$. If the capital endowment is assumed to be $\bar{K}$ then the total demand for labor would be $\bar{K} \times \mu(w)$. For each market set w, we have that μ^* is the same in all firms regardless size: we can determine total demand for labor as if it was coming from a hypothetical single firm that employs the entire supply of capital $\bar{K}$ and labor $\bar{L}$.

Figure 1.2 illustrates this process. There is a single intersection of L_S and L_d such that w^* is unique. Notice that: (i) if $w < w^* \Rightarrow L_d > L_s \Rightarrow\uparrow w \rightarrow w^*$ (production organizers will compete for workers by raising wages); and (ii) if $w > w^* \Rightarrow L_d < L_s \Rightarrow\downarrow w \rightarrow w^*$ (unemployed workers will offer to work for a lower wage). Thus this equilibrium is not only unique, but also stable if competition forces fully operate.

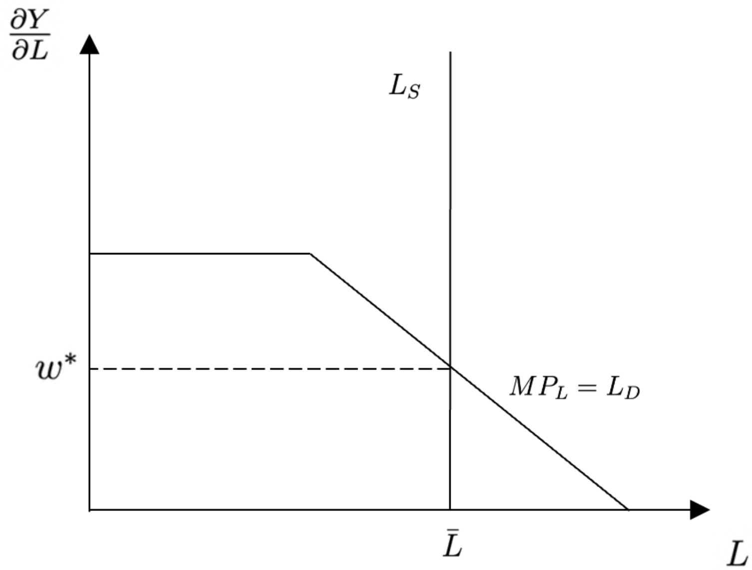

Figure 1.2: Aggregate labor demand and supply curves.

Constant returns to scale (CRS) further implies that $Y = wL + rK = MP_L \cdot L + MP_K \cdot K$. Let's see why. For a homogeneous equation of degree m we have that:

$$f(tx_1, ..., tx_n) = t^m f(x_1, ..., x_n)\ \forall t > 0$$

$$\text{let } g(t) = f(tx_1, ..., tx_n)$$

$$\Rightarrow g'(t) = \frac{d}{dt} f(tx_1, ..., tx_n) = \sum_{i=1}^{n} \frac{\partial}{\partial x_i}(tx_1, ..., tx_n) \cdot \frac{d}{dt}(tx_i) = \sum_{i=1}^{n} \frac{\partial}{\partial x_i}(tx_1, ..., tx_n) \cdot x_i$$

$$\Rightarrow g(t) = t^m f(x_1, ..., x_n) \Rightarrow g'(t) = mt^{m-1} f(x_1, ..., x_n)$$

$$\Rightarrow f(x_1, ..., x_n) = \sum_{i=1}^{n} \frac{\partial}{\partial x_i}(tx_1, ..., tx_n) \cdot x_i \text{ for } m = 1 \text{ and } t = 1$$

This implies that if one factor is paid its marginal product, the remaining product would amount to what the other factor would get if it was paid its corresponding marginal product.

So, in full employment production we would have that $Y^* = MP_L^* \cdot \bar{L} + MP_K^* \cdot \bar{K}$. This means that income distribution (in this neoclassical framework) is ahistorical, apolitical, and unaffected if the role of entrepreneurs (or capitalists) is taken by someone else: distribution is neutral in respect to the organizers of production.

If L organizes production (they hire K), then the capital market (one could obtain a demand curve for capital in the (MP_K, K) space as long as L is fixed) is the sole market where equilibrium needs to be brought by a price: $r = \frac{\partial}{\partial K} f(\bar{K}, \bar{L}) = MP_K^*$ (the price that equals the MP_K^* associated with full employment). This of course yields the same result as if production was organized by capitalists. Let's imagine that production is not organized by capitalists, but rather by price taker entrepreneurs. This agents evaluate production in respect to three prices: p, w and r (p being the price of the single commodity to be produced, assumed to be a *numéraire*). The entrepreneurs are faced with a cost identity associated to the specific amount of output they want to produce:

$$C(K, L) = wL + rK \text{ s.t. } Y = f(K, L) \text{ (costs associated to } Y)$$
$$g = pY \text{ (revenue)}$$
$$\pi = g - \min_{L,K}(C) \text{ (profits)}$$

Notice how if $g > \min_{L,K}(C) \Rightarrow \Delta Y > 0 \Rightarrow \uparrow L_d \wedge K_d \Rightarrow L \to \bar{L} \vee K \to \bar{K}$. Because of CRS entrepreneurs (if profits are positive) would up production, rising demand for both K and L, until one the factors of production becomes fully employed: guaranteeing a downward sloping demand curve for the remaining factor. Conversely, if $g < \min(C) \Rightarrow \Delta Y < 0 \Rightarrow \downarrow L_d \wedge \downarrow K_d \Rightarrow \downarrow w \wedge \downarrow r \Rightarrow \downarrow C$. This means that if profits are negative, firms lower production, lowering demand for L and K, which in turn lowers wages and profit rates: reducing costs.

In both cases demand fluctuates for both factors, which means that μ is changing until it reaches μ^* (achieving full employment). This is precisely the process of *direct factor substitution* that guarantees monotonically decreasing demand curves for L and K. Nevertheless, we can extend our understanding of this mechanism introducing the constrained cost minimization problem that entrepreneurs find, remember that firms choose an specific combination of $K, L > 0$ to produce an specific amount of Y taking w and r from the market:

$$\begin{aligned}
\min_{K,L>0} C(K,L) &= wL + rK \text{ s.t. } Y = f(K,L) \\
\mathscr{L}(K,L,\lambda) &= wL + rK + \lambda[Y - f(K,L)] \\
\Rightarrow \frac{\partial \mathscr{L}}{\partial K} &= r - \lambda MP_K = 0 \;\wedge\; \frac{\partial \mathscr{L}}{\partial L} = w - \lambda MP_L = 0 \\
\Rightarrow \lambda &= \frac{w}{MP_L} = \frac{r}{MP_K} \\
\Rightarrow MRTS_{K,L} &= \frac{MP_L}{MP_K} = \frac{w}{r}
\end{aligned}$$

Given that that our assumptions guarantee that the isoquant is smoothly, and strictly convex, the solution to the constrained cost minimization problem implies tangency between the isoquant (whose slope is $-\frac{MP_L}{MP_K}$) and the isocost (whose slope is $-\frac{w}{r}$) curves.[1] In other words: the rate at which our technology allows substitution between labor and capital equals the rate at which the market allows substitution between them (that is the factor price ratio).

Summary. The displayed tendency towards equilibrium with fully employed factors of production symmetrically occurring in both factor markets relies on the fact that, if the amount of one factor is given, the demand curve for the other factor is decreasing: the μ ratio that firms desire to use increases (demand for L goes up) when wages fall in relationship to the rate of profits. Changes in μ imply that firms can substitute a factor with the other in the production of a certain output.

1.3.2 Indirect Substitution, in Consumption

Neoclassical theory argues than in the absence of a technical factor substitution mechanism,[2] consumer choice among many different consumption goods *activates* an *indirect* factor substitution mechanism which can give place to monotonically decreasing demand curves for factors. To observe this mechanism we need some new assumptions: (i) at least two consumptions goods (γ, δ) produced in different industries (by L and K); (ii) we assume *zero* technical substitutability (fixed k). Let w and r represent monetary magnitudes, then the

[1] The isocost equation being mapped on the (L, K) space by $K = \frac{C}{r} - \frac{w}{r}L$.

[2] The economy might display a *Leontief technology*, with a so-called fixed coefficients production function of the form $Y = \min(\rho K, \chi L)$.

long-period prices of good γ and δ will satisfy:

$$p_\gamma = a_{L\gamma}w + a_{K\gamma}r$$

$$p_\delta = a_{L\delta}w + a_{K\delta}r$$

Where $a_{L\gamma}$,$a_{L\delta}$, $a_{K\gamma}$, $a_{K\delta}$, represent labor and capital inputs in both industries (also known as technical coefficients). Assume that $\frac{a_{L\gamma}}{a_{K\gamma}} > \frac{a_{L\delta}}{a_{K\delta}}$ (industry γ is more *labor intensive*). Given that labor-to-capital ratios are different in each industry, then the relative price $\frac{p_\gamma}{p_\delta}$ will depend on the $\frac{w}{r}$ ratio: $\downarrow \frac{w}{r} \Rightarrow \downarrow \frac{p_\gamma}{p_\delta}$. This means that if wages go down, relative to profits, the more labor-intensive good will become relatively cheaper in respect to the other one. This would entail (via the assumptions made in Section 1.2), that the *composition of the demand* of agents will shift in favor of good γ (maintaining the equality $\frac{p_\gamma}{p_\delta} = \frac{\partial U(\gamma,\delta)}{\partial \gamma} / \frac{\partial U(\gamma,\delta)}{\partial \delta}$).

The adaptation of production to demand requires shifting K from the δ industry to the γ industry, while increasing labor demand to maintain the fixed $\frac{a_{L\gamma}}{a_{K\gamma}}$ ratio. The same process would apply to capital. The drop (or rise) in one factor price with respect to another implies an expansion (reduction) of the sector which uses it more intensively: this is called the *indirect factor substitution mechanism.* It receives this name because the change in the composition of demand of *only* consumption goods change the k of the economy as a whole.

Chapter 2

The critique of Neoclassical Theory and the Classical Theory of Distribution

2.1 The Critique of Neoclassical Theory: A Two Sector Economy

The analysis in Subsection 1.3.2 showed that, within the neoclassical framework, even with fixed technical coefficients, *consumer substitution* across goods can indirectly generate monotonically decreasing factor-demand schedules. In this subsection we reformulate the problem from a different perspective, exploring how the distributive variables, the wage and the rate of profit are jointly determined by technology and relative prices.

Notice how Subsection 1.3.2 did not specified the process of production of K, we took K as it existed *quasi* freely in the economy without any costs. We previously assumed that capital was available without explicit production costs. We will now abandon this implicit assumption. In this economy we will assume two goods: θ and ψ. Each good can be used as an input in its own production and in the production of the other good. We can then define the techniques of production as:

$$K_{\psi\theta} + K_{\theta\theta} + L_{\theta} \rightarrow Y_{\theta}$$
$$K_{\theta\psi} + K_{\psi\psi} + L_{\psi} \rightarrow Y_{\psi} \tag{2.1}$$

Where $K_{\psi\theta}$, $K_{\theta\theta}$ and L_{θ} are the ψ, θ and L input requirements for producing a given amount of θ (symmetrically the same description applies to the ψ technique). The coefficients are fixed, implying a Leontief technology. Assuming perfect competition (so that production costs equal output price), and introducing factor prices, wages and profits, letting $p_{\theta} \equiv 1$ (so that $p_{\psi} = p$) and $v = r + \delta$ (where δ is the depreciation rate of K, such that v is the gross profit rate) we have that the wage-profit relationship, the WPR, (where $w \geq 0$ and $v \geq 0$) for a technique will be:

$$Y_{\theta} = v(pK_{\psi\theta} + K_{\theta\theta}) + wL_{\theta} \tag{2.2}$$

$$pY_{\psi} = v(pK_{\psi\psi} + K_{\theta\psi}) + wL_{\psi} \tag{2.3}$$

We can then express (2.2) and (2.3) in per-worker terms, as in Foley et al. (2019), dividing both sides of the equality by its respective labor requirements obtaining:

$$y_{\theta} = v(pk_{\psi\theta} + k_{\theta\theta}) + w \tag{2.4}$$

$$py_{\psi} = v(pk_{\psi\psi} + k_{\theta\psi}) + w \tag{2.5}$$

This implies that y_{θ} and y_{ψ} are output per worker, hence represent labor productivity in each industry. Notice that (2.2) and (2.3) can also be expressed in per-output terms, such that:

$$1 = v(pa_{\psi\theta} + a_{\theta\theta}) + wl_{\theta} \tag{2.6}$$

$$p = v(pa_{\psi\psi} + a_{\theta\psi}) + wl_{\psi} \tag{2.7}$$

Where $a_{\psi\theta}$, $a_{\theta\theta}$ and l_{θ} are the input requirements of ψ θ and L_{θ} to produce a unit of θ (notice that $L_{\theta} = l_{\theta}Y_{\theta} \Rightarrow l_{\theta} = \frac{1}{y_{\theta}}$). Similarly, $a_{\theta\psi}$, $a_{\psi\psi}$ and l_{ψ} are the input requirements of θ ψ and L_{ψ} to produce a unit of ψ. We can simplify this two sector model by assuming that the θ-good is a *pure consumption good*: which amounts to say that it is not used as an input in any production process. So, then the ψ-good becomes a *pure capital good* used in both producing itself and the consumption good.

This new assumption then implies, surely, that $k_{\theta\psi} = k_{\theta\theta} = a_{\theta\psi} = a_{\theta\theta} = 0$ (Foley et al., 2019, p. 69). We can now call $k_{\psi\theta} = k_\theta$ and $k_{\psi\psi} = k_\psi$. Also, we assume that there is no capital accumulation, such that the final product per head only consists of one good, the consumption good. With this simplification we can then solve for p and w in the equation system (2.3)-(2.4) (notice that we have *one degree* of freedom):

$$p = \frac{y_\theta}{v(k_\theta - k_\psi) + y_\psi} \tag{2.8}$$

$$w = y_\theta - pvk_\theta = y_\theta \left(1 - \frac{vk_\theta}{v(k_\theta - k_\psi) + y_\psi}\right) \tag{2.9}$$

These equations will not be linear unless $(k_\theta - k_\psi) = 0$ (hence, the capital-to-labor ratio is the same in both sectors). This would further imply that changes in distribution do not affect the relative price of capital with respect to the consumption, so that $p = \frac{y_\theta}{y_\psi}$ (the price expresses the relative productivity in the two sectors). We can calculate the first and second derivatives of w and p (for p we will only stick to the first derivative) with respect to v:

$$\frac{dw}{dv} = -\frac{y_\theta y_\psi k_\theta}{[v(k_\theta - k_\psi) + y_\psi]^2} < 0 \tag{2.10}$$

$$\frac{d^2w}{dv^2} = \frac{2y_\theta y_\psi k_\theta (k_\theta - k_\psi)}{[v(k_\theta - k_\psi) + y_\psi]^3} \tag{2.11}$$

$$\frac{dp}{dv} = -y_\theta \frac{(k_\theta - k_\psi)}{[v(k_\theta - k_\psi) + y_\psi]^2} \tag{2.12}$$

The expression $v(k_\theta - k_\psi) + y_\psi$ is always positive as long as $v \in [0, \frac{y_\psi}{k_\psi}]$. This is a direct result of solving $w = y_\theta \left(1 - \frac{vk_\theta}{v(k_\theta - k_\psi) + y_\psi}\right) \geq 0$. Another clear result that follows from differentiation, is that there exists an inverse relationship between w and v, as stated in (2.10). The signs of (2.11) and (2.12) are undefined and entirely depend on $(k_\theta - k_\psi)$. We can build a causal chain:

$$(i) \text{ If } (k_\theta - k_\psi) = 0 \Rightarrow \frac{d^2w}{dv^2} = \frac{dp}{dv} = 0 \text{ (linear WPR)}$$

$$(ii) \text{ If } (k_\theta - k_\psi) > 0 \Rightarrow \frac{d^2w}{dv^2} > 0 \wedge \frac{dp}{dv} < 0 \text{ (convex WPR)}$$

$$(iii) \text{ If } (k_\theta - k_\psi) < 0 \Rightarrow \frac{d^2w}{dv^2} < 0 \wedge \frac{dp}{dv} > 0 \text{ (concave WPR)}$$

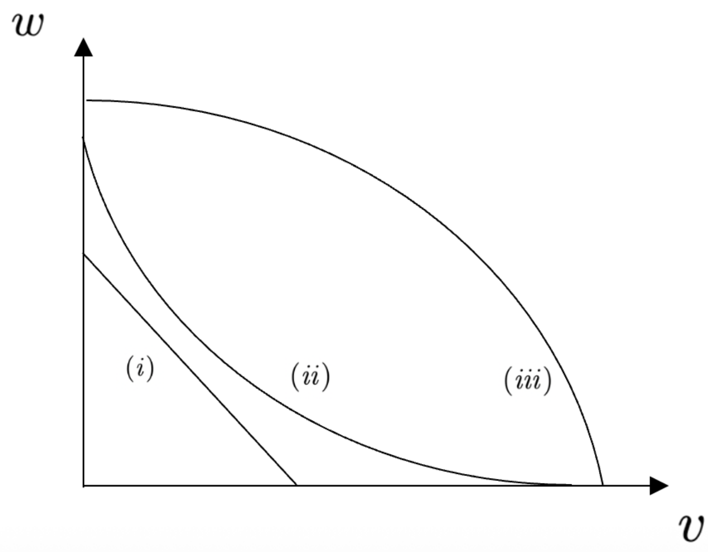

Figure 2.1: Distinct WPR according to the relative capital intensities: linear, convex and concave cases.

In Figure 2.1 we observe this three cases. In all cases we assumed a fixed capital intensities, that is k_θ and k_ψ. Nonetheless, we can see that in cases (ii) and (iii) the *value of capital per unit of labor* changes as distribution changes, this is not the case in (i). The value of capital per unit of labor is defined as $k_\theta^V = p(v)k_\theta$ and $k_\psi^V = p(v)k_\psi$. In (ii) we have a *negative price Wicksell effect*, that is, that as v rises p decreases, so that k_θ^V and k_ψ^V also decrease. This effectively describes an inverse relationship between the rate of profits and capital intensity in the economy. In (iii) we have that as v increases p also increases, entailing a direct relationship between capital intensity and the rate of profits, paradoxically: this is a *positive price Wicksell effect*.

Now, assume that we have two techniques: χ and η. By (2.9), if $v = 0 \Rightarrow y_\theta = \max(w)$ (all output goes to wages), hence the intercept of the WPR is y_θ, the product per unit of labor of the consumption good. Then, in an stationary state we will have that $pk_\theta = \frac{y_\theta - w}{v}$. The χ-technique is linear ($k_\theta^\chi = k_\psi^\chi$), while the η-technique is concave ($k_\theta^\eta < k_\psi^\eta$). Figure 2.2 also reveals that $y_\theta^\chi > y_\theta^\eta \Rightarrow k_\theta^\chi > k_\theta^\eta$. The linearity of the χ-technique implies that the value of capital per man is constant across changes in the rate of profits. Cost minimization implies that for $v \in [0, v_1) \cup [v_2, \max(v)]$ the preferred technique is χ, while at $v \in [v_1, v_2)$ the η-technique es preferred. We have two switches of techniques: at v_1 and at v_2, the second switch is called a *reswitching of techniques*.

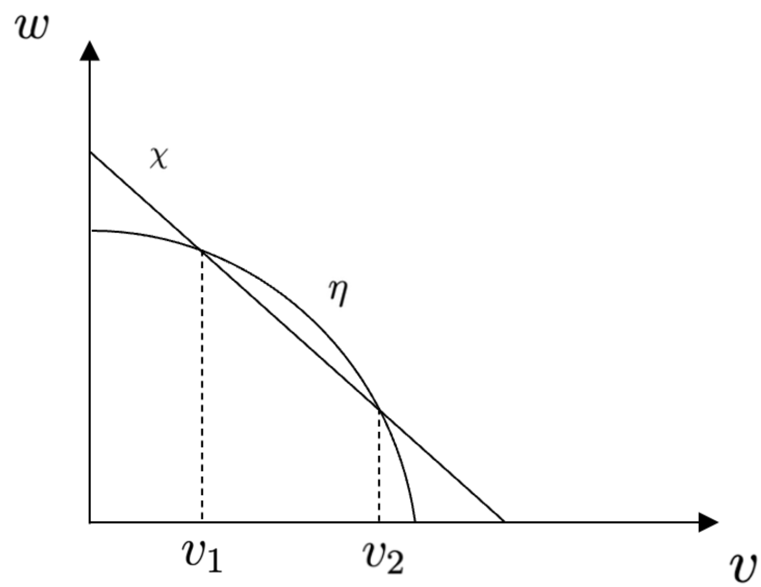

Figure 2.2: Reswitching of techniques and reverse capital deepening in the case of two distinct techniques (χ and η).

Furthermore, notice that at the switch point associated with v_2, as the rate of profit increases, we transition from a technique with less capital intensity to a technique with more capital intensity, this is called *reverse capital deepening*: there is a direct relationship between v and k. In our specific setting we have that the total Wicksell effect is defined as it follows:

$$\frac{dk_\theta^V}{dv} = \frac{dp}{dv} \cdot k_\theta + \frac{dk_\theta}{dv} \cdot p(v) \tag{2.13}$$

Where $\frac{dp}{dv} \cdot k_\theta$ refers to the price Wicksell effect and $\frac{dk_\theta}{dv} \cdot p(v)$ to the real Wicksell effects. The former are associated to the particular form that the WPR takes, while the latter are associated to changes in techniques. The χ-technique presents no price Wicksell effects, while the movement along the η-technique in the interval $v \in [v_1, v_2)$ entails a negative price Wicksell effect. Switchpoints v_1 and v_2 imply, respectively, a negative and a positive real Wicksell effect. Reswitching and reverse capital deepening show that there is no unambiguous relationship between changes in factor proportions and changes in factor prices, altering factor substitution mechanisms both in production and consumption: hence, only under extremely restrictive assumptions it is possible to derive monotonically decreasing demand curves for labor and capital.

2.2 The Analytical Structure of the Surplus Approach

The *givens* of the surplus approach are: (i) the total product of the economy and its composition; (ii) the technique (or techniques of production) and the necessary means of production (K) and labor (L); (iii) the real wage rate (w). With these assumptions the theory can determine the level of employment (indirectly), the rate of profit and the relative prices of commodities. Assume an economy producing only one good by means of labor and the same good (as circulating capital), given the available techniques for producing such commodity and given the *real subsistence wage* in terms of that commodity then the surplus P_v is:

$$Y - (K + wL) = P_V = rK + cT \tag{2.14}$$

Where P_V is the income available after reproduction to pay profits (rK) and rents (cT). If $cT = 0$, such that land is over abundant (so, free), then P_v represents profit income:

$$r = \frac{P_V}{K} \tag{2.15}$$

$$r = \frac{P_V}{(K + wL)} \text{ (if wages are advanced)} \tag{2.16}$$

Also, there is, clearly, an inverse relationship between profits and wages:

$$w = \frac{Y - (1 + r)K}{L} \tag{2.17}$$

Now, returning to the givens. (i) is not different from the one found in neoclassical analysis, there exists cost minimization. (ii) the level of social product is the result of social accumulation, Say's Law is accepted: earned income is entirely spent in consumption or investment (however, it does not entail full employment of labor). (iii) the distinct feature of the surplus approach is to take one distributive variable (usually w) as given for the determination of the surplus (and then w or r).

2.3 Wages in the Classical Approach

In the classical approach there is no intention of deriving a demand for labor inversely related to the real wage. Employment depends on the level and composition of the national product and the techniques in use. Income distribution essentially depends on the bargaining power of social classes, with a lower-bound limit defined for wages based on customary living standards and institutional limitations. This lower-bound is necessary: the absence of a decreasing relationship between wage and employment implies that complete flexible competition on the labor market could be destabilizing and destructive.

The notion of the *subsistence minimum* in the classics argues that this notion is highly social and historical (not biological), hence, different historical periods are related to different notion of what a *subsistence wage* is. Persistent changes in wages, over long periods of time, may change habits and living standard, altering societies notion of dignity, and of course, the notion of the lower-bound wage that defines the minimum purchasing power for living with dignity within an specific social stratification scheme.

Now, it is recognized that two specific conditions greatly impact the bargaining power of workers: (*i*) institutions; and (*ii*) the labor market. (*i*) When in comes to institutions, firstly, it is recognized by the classics that the separation between workers and the means of production creates a monopoly towards the access to work which gives a natural advantage in bargaining to capitalists. Secondly, specific laws prohibiting the organization of unions (while imposing no limits on the organization of trade chambers): generally speaking, the absence of equal political rights between workers and capitalists.

(*ii*) When it comes to the labor market, for the classics it was pretty clear that the employment rate affected the bargaining position of workers and capitalists, and hence the natural wage. Certain circumstances might create an advantage towards laborers (capitalists), creating competition between capitalists (laborers). In an stationary economy, however, there is a *constant scarcity of employment* opportunities: creating competition between wage earners (hence capitalists reduce wages, but never below the subsistence level). The conditions of the labor market and unemployment in the classics imply that: (*i*) there is no tendency to full employment on the demand side; (*ii*) population adjustments are slow and uncertain (discrepancies between the natural price and natural wage); (*ii*) demand for labor is a given quantity. The notion of competition in the classics is restricted by the notion of the subsistence wage, given that if a all in wages does not necessarily entail a rise in the employment rate then competition would be socially destructive. Competition is viewed as a force that equalize wages for similar types of labor. For the classics the inexistent relationship between wages and employment, substituted by the emphasis on class conflict, implies that the functional distribution of income can change in the direction in which wages change without compromising employment rates.

2.4 Analytical Problems with the Old Classical Labour Theory of Value

As seen in equation (2.14), the determination of P_V and r use two distinct aggregates that cannot be measures in physical units in a setting with more than one commodity: Y and K. If aggregates are measured in value, such measure will not be independent regarding changes in income distribution, since prices depend on the latter, as seen in Section 2.1. First attempts to solve this problem was to use *labour embodied* as an analytical tool, however, in an intertemporal setting relative prices still vary with distribution. In a two sector economy with no capital where each commodity is produced with one unit of labor, if labor inputs in one of the commodities is advanced for one period then:

$$p_1 = w$$
$$p_1 = w(1 + r)$$

Relative prices are not independent of distribution, and, here we find a tautological paradox, because prices are needed to estimate Y and K, hence P_V and r. The solution is simultaneous determination, with a given w it is possible to solve a system of price equations which simultaneously determines prices and the rate of profits (even in the presence of multiple commodities and multiple techniques of production in the WPR). So, given the (*i*) available techniques of production; (*ii*) the level and composition of the social product; and (*iii*) the real wage. The surplus approach determines the employment level, (*i*) and (*ii*), the rate of profit (inversely related to w). Changes in wages will only indirectly affect the employment level (if this changes alter consumption patterns and hence the composition of the social product) and if they do, their direction is not defined a priori in any mechanical sense. All interactions between the givens of the theory need to be analyzed with reference to a specific historical context: there is no specification of any functional relationship of general validity.

Chapter 3

The Keynesian Theory of Output and Employment

3.1 Equilibrium with Full Employment

Neoclassical theory cannot sustain that decreasing demand functions for factors are sufficient to guarantee full employment, the theory also needs to maintain that the output produced at full employment will be entirely sold. What mechanisms would ensure that at any level production output reaches and equality with aggregate demand? Note the following macroeconomic identities: $Y \equiv C + S$ and $Z \equiv C + I$ where Y is the national output, C is national consumption, S are the savings of the economy, Z is aggregate demand and I is investment.

Then $Z = Y \iff S = I$. Note that if (i) $I > S \Rightarrow Z > Y$ and (ii) $S > I \Rightarrow Y > Z$. Let $s \in (0, 1)$ be average propensity to save and Y_f the national output at full employment, then $I = sY_f = S_f$. Neoclassical theory argues that it is investment, an exogenous component of demand, that adjusts itself to any S (which in turn is a function of the level of output and the interest rate): this adjustment takes place through the effect that the interest rate has on investment (see Figure 3.1).

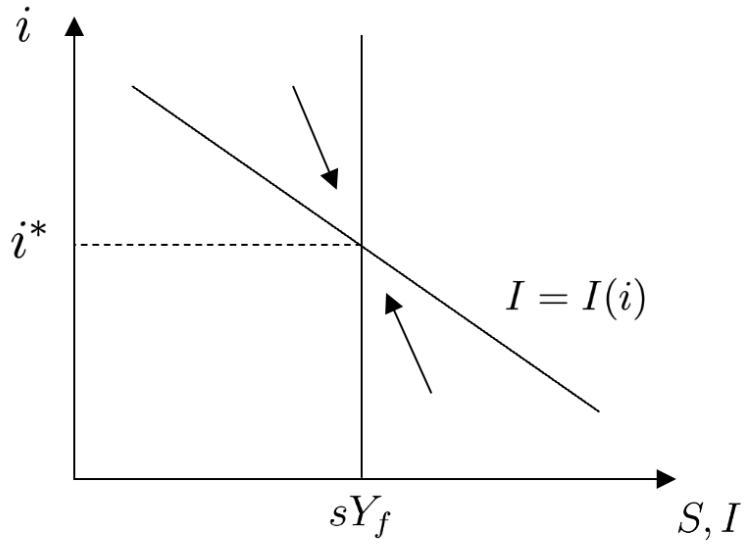

Figure 3.1: Full employment consistent tendency towards $S = I$.

Investment is understood as an inverse function of the rate of interest.

If $S_f < I$ (excess demand of loanable funds) $\Rightarrow \uparrow i \Rightarrow \downarrow I$

If $S_f > I$ (excess supply of loanable funds) $\Rightarrow \downarrow i \Rightarrow \uparrow I$

We can summarize this mechanism by this simple system of equations:

$$Y = C + S \tag{3.1}$$

$$Z = C + I \tag{3.2}$$

$$Y = Z \iff S = I \tag{3.3}$$

$$S = S(Y, i), \frac{dS}{di} > 0 \tag{3.4}$$

$$I = I(i), \frac{dI}{di} < 0 \tag{3.5}$$

How does neoclassical theory justifies this argumentation? Let $I_T^g = I_t^n + \kappa_t$ where I_t^g, I_t^n and κ_t are, respectively, gross investment, net investment and a constant of investment not associated to capital goods in a given time t. Net investment will be $I_t^n = \Delta K_t^d - \delta K_t$ where K_t^d is demand for capital of the firm and δ is the depreciation rate (note that $I_t^n \geq 0$, the change in the demand for capital must at least be enough to cover depreciation costs). It is assumed that any j-ith firm with an implicit production function $Y_j = (K_j, L_j)$ (with all the usual neoclassical properties), maximizing profits, firms will demand capital until $\frac{dY_j}{dK_j} = (i_t - \pi_t) + \delta$ where i_t and π_t are the monetary interest rate and the rate of inflation at time t, respectively. Hence $K_t^d = f(i^-)$, owing to the usual substitution mechanisms discussed in Chapter 1. Suppose that we have an i_t such that $\Delta K_t^d = K_t^d - K_{t-1}^d = \delta K_t$, if at $t+1$ then $\Delta i_{t+1} < 0$ $(i_{t+1} < i_t) \Rightarrow \Delta K_{t+1}^d > 0$ $(K_{t+1}^d > K_t^d) \Rightarrow \Delta I_{t+1}^n > 0$ (see Figure 3.1).

3.2 The Principle of Effective Demand

In keynesian theory, let's think of a simple IS-LM model without fiscal policy (no spending nor taxation) but with a central bank, we have, similarly, the following system of equations:

Commodity Market (IS)

$$Z = C + I \tag{3.6}$$

$$C = c_0 + c_1 Y \tag{3.7}$$

$$I = I_0 \tag{3.8}$$

$$Y = Z \tag{3.9}$$

Money Market (LM)

$$M_s = M_d \tag{3.10}$$

$$M_s = M_s^* \tag{3.11}$$

$$M_d = P \cdot L(Y^+, i^-) \tag{3.12}$$

Where c_0 is autonomous consumption, c_1 is the marginal propensity to consume, I_0 is the investment level exogenously determined. Solving for Y we have that:

$$Y^* = \frac{1}{1 - c_1}(c_0 + I_0) \tag{3.13}$$

Where $\frac{1}{1-c_1}$ is the *keynesian* multiplier and $c_0 + I_0$ are the components of *autonomous demand*. Let N be the economically active population and L the total number of effectively employed workers (note that $N \geq L$). Knowing that $y = \frac{Y}{L}$ is the average productivity per worker, we have that

$$L^* = \frac{Y^*}{y} = \left(\frac{1}{y}\right)\left[\frac{1}{1-c_1}(c_0 + I_0)\right] \tag{3.14}$$

$$u^* = \frac{N - L^*}{N} \in [0, 1] \tag{3.15}$$

So, given c_1, c_0 and I_0 (the marginal propensity to consume and autonomous demand), there is only one level of output Y^* that ensures the macroeconomic equilibrium between supply and demand, such that $S = I$. We can see this relationship in Figure 3.2.

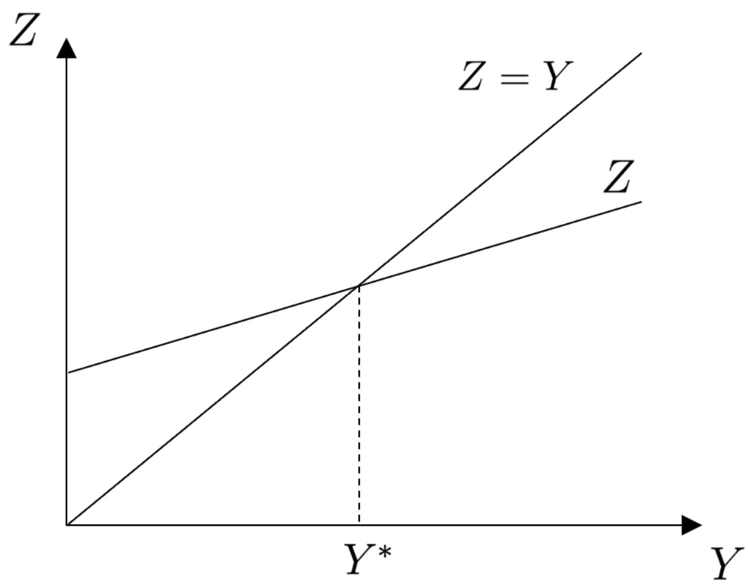

Figure 3.2: The *keynesian asps*: the principle of effective demand.

Notice two things: (i) the causality runs from Z towards Y, such that savings are determined by investments (and not vice-versa, as in neoclassical theory); and (ii) there is no indication whatsoever around an equality (or equality achieving mechanism) between the equilibrium output Y^* and full employment output Y_f. Notice the mechanism:

$$\text{If } Z > Y \Rightarrow \uparrow Y \text{ (firms increase production) } \Rightarrow \uparrow S = sY \Rightarrow S \to I$$

$$\text{If } Z < Y \Rightarrow \downarrow Y \text{ (firms decrease production) } \Rightarrow \downarrow S = sY \Rightarrow S \to I$$

Knowing $s \in (0, 1)$, and knowing that $Y = C + S$ we can build a savings function:

$$S = (-c_0) + (1 - c_1)Y \tag{3.16}$$

This function as a negative intercept $-c_0$, which implies that when $c_0 = 0$ we have that $S = (1 - c_1)Y = sY \Rightarrow s = (1 - c_1)$. Movements in c_0 shift the savings curve downwards maintaining equilibrium between savings and investments (while elevating output), while changes in investments imply movements along the curve altering the level of aggregate savings and income equilibrium. Nonetheless note that in this functional definition we encounter a *paradoxical* result: if the marginal propensity to save increases then we observe a fall of aggregate income while leaving aggregate savings unchanged (equal to investments). This latter phenomena is called the *paradox thrift*, it holds given that $c_1 = 1 - s$ and $\frac{dY^*}{dc_1} = \frac{1}{(1-c_1)^2}(c_0 + I_0) > 0$. **Summary**. The principle of effective demand argues that aggregate demand (Z) determines the equilibrium output (Y^*), and such L^* and u^*, where $S = I$.

3.2.1 The case of Flexible Wages and Prices

Let w_n be the nominal wage, $w_r = \frac{w}{P}$ the real wage and let P the general level of prices. Assume that the economy is an equilibrium Y^* such that $u^* > 0$ ($N - L^* > 0$), and assume that labor is the only input of production, or conversely that in the short-run variable costs depend only on labor, such that $c_{mg} = \frac{\partial VC}{\partial Y}$. Will a decrease in nominal wages affect positively the level of employment? Given that $Y = f(c_1, c_0, I_0)$ then a fall in nominal wages has to affect one of the three variables determining equilibrium output (and hence the equilibrium employment). Let $l = \frac{1}{y} = \frac{Y}{L}$. We have that the price equation is

$$p = w_n l \tag{3.17}$$

$$c_{mg} = w_n \frac{dL}{dY} = w_n \left(\frac{1}{MP_L} \right) \tag{3.18}$$

Let $x = f(t)$, where t is time. The following terminology will mostly be relevant when dealing with growth-change rate inequalities. Then, in discrete and continuous terms we can define their change rate change and the growth rate as:

$$(i)\ \dot{x} = \Delta x = x_t - x_{t-1} \text{ and } \hat{x} = \frac{\dot{x}}{x} \text{ (discrete variables)}$$

$$(ii)\ \dot{x} = \Delta x = \frac{dx}{dt} \text{ and } \hat{x} = \frac{dx}{dt} \cdot \frac{1}{x} = \frac{d\ln(x)}{dt} \text{ (continuous variables)}$$

Then we can construct a casual chain for what might occur to equilibrium output and other related variables in the case of a reduction w_n. Let $c_1 = f(c_w, c_r, \omega, 1-\omega)$, which in turn says that the marginal propensity to consume in the economy is a function of the marginal propensity to consume of wage (c_w) and profit (c_r) earners, and the wage and profit share: $\omega = \frac{wL}{Y} = \frac{w}{y}$ and $1-\omega = \frac{Y-wL}{Y} = 1-\frac{w}{y}$. And finally, let $c_w > c_r$. Such that $\hat{w}_n = \lambda < 0$:

$$(i) \text{ If } p = c_{mg} \Rightarrow \hat{w}_n = \hat{p} = \lambda < 0 \Rightarrow\downarrow M_d \Rightarrow\downarrow i \text{ and if } I = f(i^-) \Rightarrow\uparrow I \wedge \uparrow Z \Rightarrow\uparrow Y^* \wedge \uparrow L^*$$

$$(ii) \text{ If } p \neq c_{mg} \ \wedge |\hat{w}_n| > |\hat{p}| \Rightarrow \hat{w}_r < 0 \Rightarrow\downarrow c_1 \Rightarrow\downarrow Y^* \wedge \downarrow L^*$$

Mechanism (ii) displays a casual chain in which, if nominal wages fall *faster* than prices, we observe a distributional shift of income towards profit earners, and since $c_w > c_r$, then the marginal propensity to consume of the economy also falls: reducing the term $\frac{1}{1-c_1}$ and hence output equilibrium. Mechanism (i) is known as the *Keynes effect*. In which a proportional fall in nominal wages and prices (leaving real wages constant) reduces nominal money demand (or, also equivalently, increases real money supply $\frac{M_s}{P} = L(Y, i)$). Equilibrium in the money market then implies a fall in the rate of interest. If $\frac{dI}{di} < 0$ then we could expect a positive change in investment, which in turn would elevate aggregate demand, output and employment equilibrium. The latter increase in L^* would also imply a fall in MP_L, hence also a fall in p and w_r.

Two main caveats can be found in this latter mechanism. (i) let i_{lt} be the rate of interest such that $\lim_{i \to i_{lt}} \frac{\partial L}{\partial i} = -\infty$ (the liquidity preference function becomes infinitely elastic with respect to the interest rate), then, if $i = i_{lt}$, all additional liquidity generated by the rise in the real money supply $\frac{M_s}{P}$ will be demanded by the public: there is no interest rate channel that increases investment (and hence Z, Y^* and L^*). (ii) if $\frac{dI}{di} \approx 0$, such that investment is a function of expectations, *animal spirits* and so forth, will then changes in i would have no effect on Z, Y^* and L^*. In both scenarios, at different links, this casual chain is broken:

$$\hat{P} < 0 \Rightarrow\uparrow \frac{M_s}{P}(\text{ or } \downarrow M_d) \Rightarrow\downarrow i \Rightarrow\uparrow I \wedge \uparrow Z \Rightarrow\uparrow Y^* \wedge \uparrow L^*$$

Keynes never rejected the idea of an inverse relationship between real wages and employment, nor did he explicitly rejected an inverse relationship between investment and the interest rate. Hence, the principle of effective demand, that being that Z determines an equilibrium output Y^* appears to be only valid in the case where prices and money wages are rigid, the money market finds itself in liquidity trap conditions, and, in short-run conditions: the Keynes effect still operated a causality from Z to Y in the short-run, even though it is Z which is adjusting via changes in prices towards a full employment equilibrium Y_f.

3.3 The State of the Art: Contemporary Macroeconomics

The critique of neoclassical theory revised in Section 2.1 implies a rejection of the inverse relationship between the interest and investment (such that the IS curve is vertical), and, a rejection of the inverse relationship between real wage and the employment level. After Keynes's publication of *The General Theory of Employment, Interest and Money*, the failure to successfully reconcile the marshallian assumptions he accepted with his general theory of output and employment (that being, the principle of effective demand), led to the development of the $IS - LM$ model: which, with nominal flexible wages and prices ensures a tendency towards full employment (relegating the role of aggregate demand to matters related to the business cycle).

3.3.1 *Mainstream* Theory

Since the late 1970's macroeconomic theory saw shift towards monetarism, rational expectations theory and business cycle models. Previously, in the post-war period (1950's-1970's), discretional economic policies (counter-cyclical), particularly fiscal, where widely admitted. This stopped to be the case in the late 1970's, where in large policy authorities started to put a greater emphasis on monetary policy/interest rate policy.

The 1980's and 90's saw the birth of new-keynesian/new consensus models, in part owing to the failure of monetarist monetary policy. In practice, this was a return towards more pragmatic and flexible $IS - LM$ inspired counter-cyclical policies. Microfoundations where introduce into macroeconomic models in order to explain rigidities, the role aggregate demand was again highlighted for the short-run: nominal and real rigidities might explain a certain level of equilibrium unemployment (see the concept of NAIRU, that is, the *Non-Accelerating Inflation Rate of Unemployment*). In the long-run, economic growth occurs along an equilibrium path of full employment (or maximum employment, dependent of competition, regulation) determined by supply-side factors.[1]

3.3.2 *Non-mainstream* Theory

The first generations of post-keynesian authors accepted Keynes's conclusions regarding effective demand, plus the weakness or nonexistence of interest rate based adjustment mechanisms towards full employment. The two main foundational pillars of post-keynesian economics are: (*i*) the acceptance of the principle of effective demand as the theory of output and employment; and (*ii*) an institutional-political-historical explanation of income distribution (abandoning the notion of monotonically decreasing demand functions for labor, capital and investment). Other commonly shared positions within this approach are: (*i*) the notion of money supply as an endogenous variable (hence accepting that the central bank can in general control the structure of interest rates); and (*ii*) the abandonment of *methodological individualism*.

Keynes's propositions in the *General Theory* are to be understood as *general* theories of employment and output, and not as theories of depressions, short-run conditions or cyclical fluctuations. Hence, the aim is to prove that below-full-employment equilibrium positions are possible and persistent. It has been contended that Keynes only succeeded partially in this respect. However, the simple model described in Section 3.2 is very rich in relation to neoclassical theory: (*i*) the equilibrium between Z and Y, emerging from market interactions, need not be a full employment equilibrium; (*ii*) investments determine savings (in conjunction with the other components of autonomous demand); and (*iii*) that the level of employment L^* and w depends on effective demand.

As we previously discussed in Subsection 3.2.1 the neoclassical synthesis took advan-

[1]Some, not very popular, new-keynesian models recognized, particularly after the 2008 global economic crisis, the possibility that recessions may cause a permanent increase in the NAIRU and in potential output: this phenomena is called *hysteresis*.

taged of the fact that Keynes reached a-neoclassical conclusions without rejecting marshallian foundations (accepting decreasing demand curves for labor, capital and investment). The so called *Keynes effect* became the foundation of the neoclassical synthesis and most mainstream varieties of macroeconomic models since then (saving the notion of an embedded tendency towards full employment in the construction of the principle of effective demand). The arguments made by non-mainstream economists (liquidity trap conditions, price-wage rigidities and the inelastic interest-investment relationship) where disregarded by mainstream economists as *ad hoc arguments* that would only hold in the short-run. Thus, the impact of the interest rate on investment (and hence on Z) is a crucial dividing point between mainstream and non-mainstream economic theory.

Some post-keynesian responses towards the neoclassical synthesis can be categorized as coming from a *fundamentalist* strand of thinkers trying to defend the principle of effective demand on the same grounds as Keynes did. They accept an investment function negatively related to the interest rate, though regarding this functions as unstable (because of *animal spirits, financial instability and expectations*). They vindicate the short-run as the primarily framework of economic analysis, abandoning the classical and neoclassical idea of long-period tendencies and the so called *normal positions*. The *fundamentalists* have provided important insights regarding the functioning of financial markets, and the interaction that these markets may have with the real economy. They diverge from other post-keynesian strands of economic thought in two main aspects: (*i*) the idea that central banks *can* control the structure of interest rates (through the control of short-term interest rates); and (*ii*) the extension of the principle of effective demand to the long-run.

We can recognize another, rather heterogeneous, strand of post-keynesian authors (Sraffians, Kaleckians, Kaldorians). These authors generally accept the principle of effective demand while abandoning (on the basis of the *Cambridge Capital Controversies*) the marshallian elements present in Keynes's work. It is argued that, empirically, the self-adjusting mechanisms proposed by the neoclassical synthesis are unobservable: hence it is possible to use the principle of effective demand as a determinant of long-run levels of output and employment. This opened the subject of studying economic growth through changes in autonomous demand, and particularly, investment. Four main lines have emerged concerning the determinants of investment: (*i*) *animal spirits* and long-term expectations; (*ii*) current and/or expected profits; (*iii*) fully/partially explained by income distribution and the autonomous components of demand; and (*iv*) *schumpeterian* competition in technical innovation.

Chapter 4

Growth Models

4.1 Harrod-Domar

This exposition follows some of the elements put forward by Thirlwall (2017, pp. 103-109) This model can be understood as a dynamic extension of Keynes's *General Theory*, in order for an economy to be in equilibrium we need that $S = I$ (see 3.2).[1] What are the conditions in which this equality will hold over time? If changes in income induce investment, what must be the rate of growth of income for plans to invest to equal plans to save (thus assuring a moving equilibrium)? Noting the following accounting identities hold (in a world with no capital depreciation):

$$S = I \tag{4.1}$$

$$S = sY \tag{4.2}$$

$$\dot{K} = I \tag{4.3}$$

$$v_a = \frac{K}{Y} = \frac{\dot{K}}{\dot{Y}} \tag{4.4}$$

$$\Rightarrow I = v_a \dot{Y} \tag{4.5}$$

$$\Rightarrow g_a = \hat{Y} = \frac{I}{v_a Y} = \frac{sY}{v_a Y} = \frac{s}{v_a} \tag{4.6}$$

Where s is the propensity to save, and v_a is *the actual* (observed) capital-output ratio, which is assumed constant over time. Then, equation (4.6) only reflects an accounting

[1] Note that we will use the same growth-change rate notation as the one defined in Subsection 3.2.1.

identity inherited from the principle of effective demand ($S = I$) we call this equation the *actual growth rate*. Important observation: *nothing guarantees* that the actual-observed growth rate is consistent with full employment of capital and labor. Now we can define the following set of equations:

$$S = sY \tag{4.7}$$

$$\alpha = \frac{\dot{K}_r}{\dot{Y}} = \frac{I}{\dot{Y}} = \frac{K_r}{Y} = v_n \tag{4.8}$$

$$I = \alpha\dot{Y} \tag{4.9}$$

$$\text{If } sY = \alpha\dot{Y} \tag{4.10}$$

$$\Rightarrow g_w = \hat{Y} = \frac{s}{\alpha} = \frac{s}{v_n} \tag{4.11}$$

Where equation (4.7) is the Keynesian savings function, such that s is the propensity to save, thus, this equation gives the potential supply of investment goods. The demand for investment, defined in equation (4.9), is given by the *acceleration principle*. The accelerator coefficient α is defined in equation (4.8), this coefficient is measured as the *required* amount of investment (or extra capital) to produce a unit flow of output: constant over time, it's implicit that it expresses the *normal* (or required) capital-output ratio. If planned savings (sY) equal planned investment ($\alpha\dot{Y}$), then the required rate of growth for a *moving equilibrium* is the warranted rate of growth g_w (this is the rate consistent with normal capacity utilization of capital). Note that:

(i) If $g_a > g_w \Rightarrow v_a < v_n$ (deficit of K, over-utilization) $\Rightarrow\uparrow I \Rightarrow\uparrow g_a$

(ii) If $g_a < g_w \Rightarrow v_a > v_n$ (surplus of K, under-utilization) $\Rightarrow\downarrow I \Rightarrow\downarrow g_a$

Both scenarios describe a divergence if the system parts from a situation where $g_a \neq g_w$. The assumed constancy of capital, and the explicit causal direction that goes from investment towards the growth of output guarantee this path of widening disequilibrium. This is the so called *Harrodian instability*. Plus, exogenously determined, we can define the natural rate of growth as:

$$g_n = n + \hat{y} \tag{4.12}$$

Where n is the growth rate of the labor force ($\hat{L} = n$) and $\hat{y}$ is the growth rate of labor productivity (output per worker). The natural rate of growth defines the long-run full employment equilibrium rate, also, it sets an upper limit to the actual growth rate, if $g_a > g_w \Rightarrow g_a \rightarrow g_n$. Of course, it is not possible that $g_a > g_n$. Thus, the full employment of labor and capital requires:

$$g_a = g_w = g_n \tag{4.13}$$

The actual growth rate depends on realized saving and capital-output ratio, the warranted rate depends on desired accumulation plans, and the natural rate depends on demographic and technological factors. It is only if, by chance, this equality is achieved, that the economy moves along a balanced full-employment growth path. This is the so called *knife edge equilibrium*. Observe how there is nothing on the model that would automatically generate this result.

4.2 Solow-Swan

Following Barro and Sala-i Martin (2004, p. 23-33), we have these basic assumptions: (*i*) The economy is a one-sector production technology in which output is an homogeneous good both consumed C or invested I, capital goods are homogeneous and they depreciate at a constant rate $\delta > 0$. (*ii*) A closed economy with no government purchases, with a constant average savings rate $s \in [0, 1]$ $sY = S = Y - C = I \Rightarrow c_m = (1 - s)$ ($s = \frac{S}{Y}$ and $c_m = \frac{C}{Y}$) (where c_m is the average propensity to consume). (*iii*) The labor force grows at a constant exogenous growth rate $\hat{L} = n$. (*iv*) The net increase in the stock of capital goods is given by this equation:

$$\dot{K} = I - \delta K = sY - \delta K \tag{4.14}$$

And, finally, (*v*) the economy's production is characterized by a neoclassical production function, $f : \mathbb{R}^2_+ \rightarrow \mathbb{R}^+$, $Y = f(K, L)$ that satisfies: (*1*) $tY = tf(K, L) = f(tK, tL)\ \forall t > 0$ (constant returns to scale CRS, homogeneity of degree one); (*2*) $MP_L = \frac{\partial Y}{\partial L} > 0 \wedge MP_K = \frac{\partial Y}{\partial K} > 0, \frac{\partial^2 Y}{\partial L^2} < 0 \wedge \frac{\partial^2 Y}{\partial K^2} < 0$ (decreasing marginal productivity for L and K); and (*3*) the *Inada Conditions*: $\lim_{K \rightarrow 0} \left(\frac{\partial Y}{\partial K}\right) = \lim_{L \rightarrow 0} \left(\frac{\partial Y}{\partial L}\right) = \infty$ and $\lim_{K \rightarrow \infty} \left(\frac{\partial Y}{\partial K}\right) = \lim_{L \rightarrow \infty} \left(\frac{\partial Y}{\partial L}\right) = 0$. Remembering that $k \equiv \frac{K}{L}$ and $y \equiv \frac{Y}{L}$, and setting $\lambda = \frac{1}{L}$, then, because of CRS

($\lambda Y = f(\lambda K, \lambda L)$) we have an *intensive production function*:

$$y = f(k) = f(k, 1) \tag{4.15}$$

Equation (4.15) is graphed in Figure 4.1. To each (k,y) combination there is a unique, but not constant, capital-output ratio v. The slope associated to point b is $\rho_b = \frac{y_b}{k_b} = \frac{1}{v_b}$, the one associated to point a is $\rho_a = \frac{y_a}{k_a} = \frac{1}{v_a}$. Notice that $\rho_b < \rho_a \iff v_a < v_b$. Following the Harrod-Domar model (see Section 4.1), and particularly equations (4.11) and (4.12), we can see that:

$$g_w = \frac{s}{v_b} < g_n = \frac{s}{v_a} \Rightarrow L^d < L^s \Rightarrow\downarrow w \Rightarrow k_b \to k_a(v_b \to v_a) \Rightarrow g_w = g_n = \frac{s}{v_a}$$
$$g_w = \frac{s}{v_a} > g_n = \frac{s}{v_b} \Rightarrow L^d > L^s \Rightarrow\uparrow w \Rightarrow k_a \to k_b(v_a \to v_b) \Rightarrow g_w = g_n = \frac{s}{v_b}$$

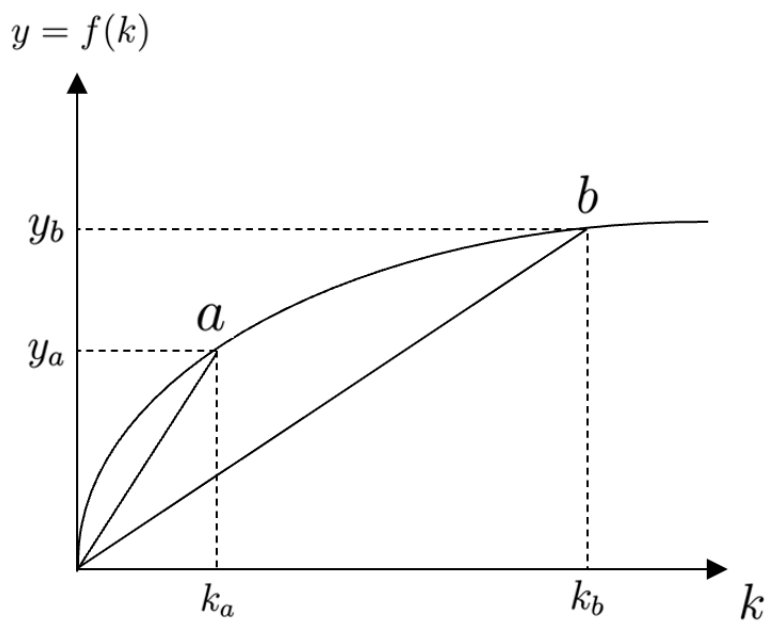

Figure 4.1: The neoclassical intensive production function and its adjustment mechanisms.

Hence, by construction, the Solow-Swan growth model avoids the problems of *harrodian instability* and of the *knife edge equilibrium* ensuring that, through direct substitution in production, the capital-output ratio always converges with the one required for full employment of labor and capital. We can now follow to identify the conditions under which capital intensity increases or decreases. The change in K is given by (4.14), dividing both sides of the equation by L we get:

$$\frac{\dot{K}}{L} = sf(k) - \delta k \tag{4.16}$$

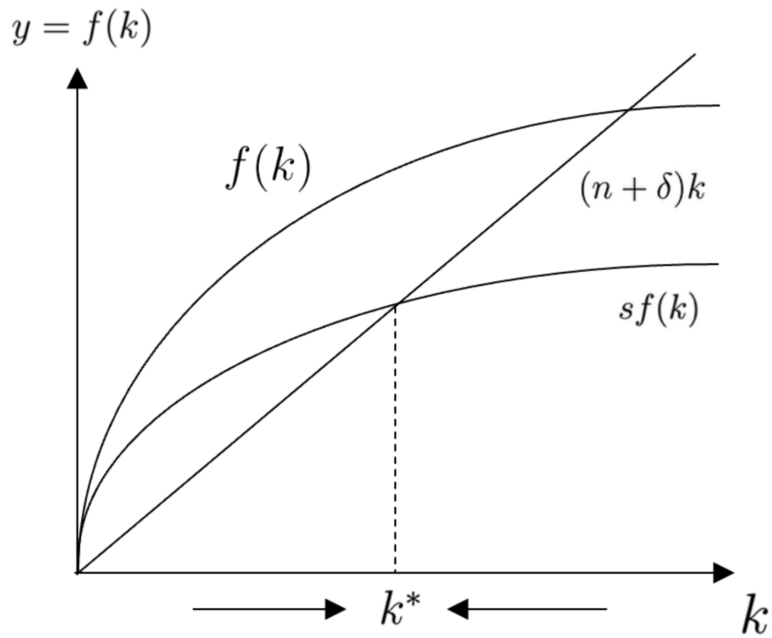

Figure 4.2: The Solow-Swan model and the steady state.

This last expression is would appear to be a not an easily solvable ordinary differential equation, the right-hand side of the equation is expressed in per capita variables only, while the left-hand side doesn't. Remembering that all variables are a function of time, we can take the derivative of $k \equiv \frac{K}{L}$ with respect to this variable, using the quotient rule we have:

$$\begin{aligned}
\dot{k} &= \frac{\dot{K}L - \dot{L}K}{L^2} = \frac{\dot{K}}{L} - \frac{\dot{L}}{L} \cdot \frac{K}{L} = \frac{\dot{K}}{L} - nk \\
\Rightarrow \frac{\dot{K}}{L} &= \dot{k} + nk \\
\Rightarrow \dot{k} + nk &= sf(k) - \delta k \\
\Rightarrow \dot{k} &= sf(k) - (n+\delta)k \qquad (4.17)
\end{aligned}$$

Equation (4.17) is the fundamental equation of the model. It describes that capital intensity (k) change rate is positive (negative) when $sf(k) > (n+\delta)k$ ($sf(k) < (n+\delta)k$).

That is, the difference between the savings curve (or gross investment per unit of labor) and the depreciation curve (or the effective depreciation of k). From this result it is obvious that If $s = 0 \Rightarrow \dot{k} < 0$. Notice that if $sf(k^*) = (n + \delta)k^* \Rightarrow \dot{k} = 0$. This called a *steady state position*, see Figure 4.2.

The corresponding value of k that satisfies $\dot{k} = 0$ is $k^* = \frac{sy^*}{(n+\delta)}$. It is only because of k^* solving $sf(k^*) = (n+\delta)k^*$ that $y^* = f(k^*)$, let c be the consumption per capita ($c = \frac{C}{L}$), since k is constant in the steady state, then consumption per capita (notice that $c = (1 - s)y^*$) and product per capita will also be constant values in the steady state (which means that $\hat{Y} = \hat{K} = \hat{C} = n$). We can analyze the dynamics, if $k < k^* \Rightarrow sf(k) > (n + \delta)k \Rightarrow \dot{k} > 0$; if $k > k^* \Rightarrow sf(k) < (n + \delta)k \Rightarrow \dot{k} < 0$. Trajectories converge to a unique k^* given: (i) concavity; (ii) CRS; and (iii) *Inada Conditions*.

4.3 The Cambridge Equation Model

Neoclassical theory dealt with the problem of *harrodian instability* through the introduction of neoclassical factor substitution mechanisms (see Section 4.2). In the *Cambridge Equation Model*, this problems are dealt through a different adjustment mechanism, concerned with how changes in income distribution might induce a path of convergence between the actual growth rate the warranted growth rate.

The model has the following assumptions: (i) investment (I) is exogenously determined; (ii) the average propensity to save of wage earners is lower than the one from profit earners $(s_w < s_r)$; and (iii) the national output is $Y = Y^*$, such that $\frac{N - L^*(Y^*)}{N} = 0$ and $u = \frac{Y}{Y^*} = 1$ where u is the *degree of utilization of productive capacity* (full employment of labor and capital). Then, we can define the following system of equations:

$$S = I \tag{4.18}$$

$$\dot{K} = I \tag{4.19}$$

$$Y \equiv wL + rK = W + R \tag{4.20}$$

$$S = s_w W + s_r R \tag{4.21}$$

$$s = s_w \omega + s_r(1 - \omega), \text{ where } \omega = \frac{wL}{y} = \frac{W}{L} \tag{4.22}$$

Where W and R are the total wage bill and profits of the economy, thus ω is the wage share such that $1-\omega = 1-(\frac{W}{Y}) = 1-\frac{Y-R}{Y} = \frac{R}{Y} = \frac{rK}{Y} = rv$ (notice that w is the real wage). Since $I = S$ and $W = Y - R$, we can manipulate equation (4.21) obtain the next equation

$$I = s_w(Y-R) + s_r R = (s_r - s_w)R + s_w Y \tag{4.23}$$

$$R = \frac{1}{(s_r - s_w)}I - \frac{s_w}{(s_r - s_w)}Y \tag{4.24}$$

Under the further assumption that $s_w = 0$, we can then multiply both sides of the equality by $\frac{1}{K}$, finally reaching the following expression:

$$\frac{R}{K} = \frac{1}{s_r}\frac{I}{K}$$

$$\Rightarrow g_k = s_r r = s_r r_n \tag{4.25}$$

Where r_n is the rate of profits consistent with normal capacity utilization. Given our assumptions, g_k is exogenously determined by capitalists *animal spirits*, such that changes in g_k will necessarily imply changes in r, and thus, a fall (a rise) in the wage share (profit share), which can also be defined as $\omega = \frac{Y-R}{Y} = 1 - rv$ ($\pi = rv$). What causes the fall in real wages? In a state of normal capacity utilization. We can draw the following causal chain:

$$I_0 < I_1 \Rightarrow Y_0 = Z_0 < Z_1 \Rightarrow P_0 < P_1 \Rightarrow \downarrow w \Rightarrow \downarrow \omega \Rightarrow (\uparrow \pi) \wedge \uparrow r \Rightarrow \uparrow s = s_r \pi$$

An increase in investment causes an excess of aggregate demand (Z), rising the general level of prices (P), thus reducing the real wage rate and the wage share (rising the profit share and the profit rate) which ultimately causes an increase in the average propensity to save (s). Hence the principle of effective demand determines output in the short-run and income distribution in the long-run. Notice that

$$g_k = s_r r_n = s_r \frac{R}{K} = s_r \frac{R/Y^*}{K/Y^*} = \frac{s_r \pi}{v_n} = \frac{s}{v_n} = g_w$$

The Harrod-Domar warranted growth equation (4.11) is an equilibrium condition necessary in order for the economy to grow along a path that guarantees $S = I$, it's assumed static. In the Cambridge Equation Model of growth this is no longer the case. As we previously stated, if $\dot{g}_k > 0 \Rightarrow \uparrow r_n(\uparrow \pi) \Rightarrow \uparrow s \Rightarrow g_w \rightarrow g_k$ (this implies that the model is stable). However, this conclusions entirely depend on the fact that it is assumed that the economy parts from a state of normal capacity utilization. We can multiply both sides of equation (4.20) by $\frac{1}{Y}$ and the right hand side of the equation by $\frac{Y^*}{Y^*}$ to obtain, and let $l_n = \frac{L}{Y^*}$ the desired labor-output ratio, then:

$$1 = w\frac{L}{Y^*}\frac{Y^*}{Y} + r\frac{K}{Y^*}\frac{Y^*}{Y} = \frac{wl_n + rv_n}{u}$$

$$\Rightarrow r = \frac{u - wl_n}{v_n}, \text{ where } \frac{\partial r}{\partial u} = \frac{1}{v_n} > 0 \tag{4.26}$$

$$\Rightarrow w = \frac{u - rv_n}{l_n}, \text{ where } \frac{\partial w}{\partial u} = \frac{1}{l_n} > 0 \tag{4.27}$$

This implies that profits (r) and wages (w) are in itself dependent of the actual degree of utilization, and can move in the same direction as the economy approaches full employment, which in turn doesn't allow the necessary compression of the wage share that implies a rise in the average propensity to save s (the mechanism that adjusts g_w to g_k).

4.4 Neo-Kaleckian

Following Lavoie (2022, pp. 383-387), this *canonical* representation of this growth model consists of three equations:

$$g_s = s_r r \tag{4.28}$$

$$g_i = \alpha + \beta(u^e - u_n) \tag{4.29}$$

$$r_{PC} = r = \frac{\pi}{v_n}u \tag{4.30}$$

Where equation (4.28) describes the the rate of growth permitted by capacity saving as a function of the average propensity to save of profit earners (s_r) and the actual profit rate ($r = r_{PC}$) seen from the cost side, it is exactly the same as the fundamental expression (4.25) in the Cambridge Equation Model (see Section 4.3). Equation (4.29) describes the growth of capital stock as a function of the firms expected long-term growth of sales (α) and the capacity utilization gap between expected (or actual) utilization and normal utilization ($u^e - u_n$), hence if $u^e = u_n \Rightarrow g_i = \alpha$ and if $u^e < u_n$ ($u^e > u_n$) $\Rightarrow g_i < \alpha$ ($g_i > \alpha$). It is assumed that each firm strives to return to normal capacity utilization.

Finally, equation (4.30) describes the actual profit rate as a share of the profit share ($\pi = \frac{R}{Y}$), the normal capital-to-output ratio $v_n = \frac{K}{Y^*}$ and the actual degree of capacity utilization

($u = \frac{Y}{Y^*}$). This is derived noting that $r = \frac{R}{K} = (\frac{R}{K}) \cdot (\frac{Y}{Y}) \cdot (\frac{Y^*}{Y^*}) = (\frac{R}{Y}) \cdot (\frac{Y^*}{K}) \cdot (\frac{Y}{Y^*}) = \frac{\pi}{v_n} u$. We can know identify the equilibrium equation using two distinct paths, one related to the rate of profit and one related to the savings equation. Assuming $g_i = g_s$ (for the first equation) and substituting (4.30) into (4.28) (for the second equation) we get:

$$r_{ED} = \frac{\alpha + \beta(u - u_n)}{s_r} \tag{4.31}$$

$$g_s = \frac{s_r \pi}{v_n} u \tag{4.32}$$

Equation (4.31) is *the effective demand constrained profit rate*, which represents the locus of all equilibrium points where $g_i = g_s$ (where all produced goods are sold). Note that $g_i = g_s \Rightarrow u^e = u$ In the second expression (4.32) we get a savings equation as a function of the rate of capacity utilization. We can either equalize (4.29) and (4.32) (which assumes that in the long-run $g_i = g_s$) or (4.30) and (4.31), to obtain the equilibrium rate of utilization:

$$u^* = u = \frac{\alpha - \beta u_n}{(s_r \pi / v_n) - \beta} = \frac{(\alpha - \beta u_n) v_n}{s_r \pi - \beta v_n} \tag{4.33}$$

Equation (4.32) defines u as the endogenous variable that brings growth of capacity savings into line with the rate of growth of the capital stock: this is called the *keynesian hypothesis* or *keynesian stability condition*. This as long as $\frac{s_r \pi}{v_n} > \beta$ or $s_r \pi > \beta v_n$, which implies that the slope of the g_s curve is higher than the g_i curve: g_s is more *sensitive* (elastic) to changes in the actual rate of capacity utilization u, than g_i is. This stability conditions also implies that the slope of the r_{ED} relationship is smaller than the one found on the r_{PC} relationship. The adjustment mechanism can be written as:

$$\dot{u} = \mu(g_i - g_s), \text{ for } \mu > 0 \tag{4.34}$$

Firms will adjust the level of output to the disequilibrium observed in the goods market, thus, they will increase (decrease) output and hence increase (decrease) capacity utilization whenever aggregate demand exceeds aggregate supply. If $g_i < g_s \Rightarrow \dot{u} < 0$ and vice-versa. Assume that $\dot{\pi} < 0$, then both the g_s and r_{PC} curves will shift downwards, as in Figure 4.3: the new curves are highlighted in red. The mechanics are the following:

$$\dot{\pi} > 0 \Rightarrow u^* \to u^K (u^K > u^*) \Rightarrow u^K \to u^{**} \wedge r^* \to r^{**} \tag{4.35}$$

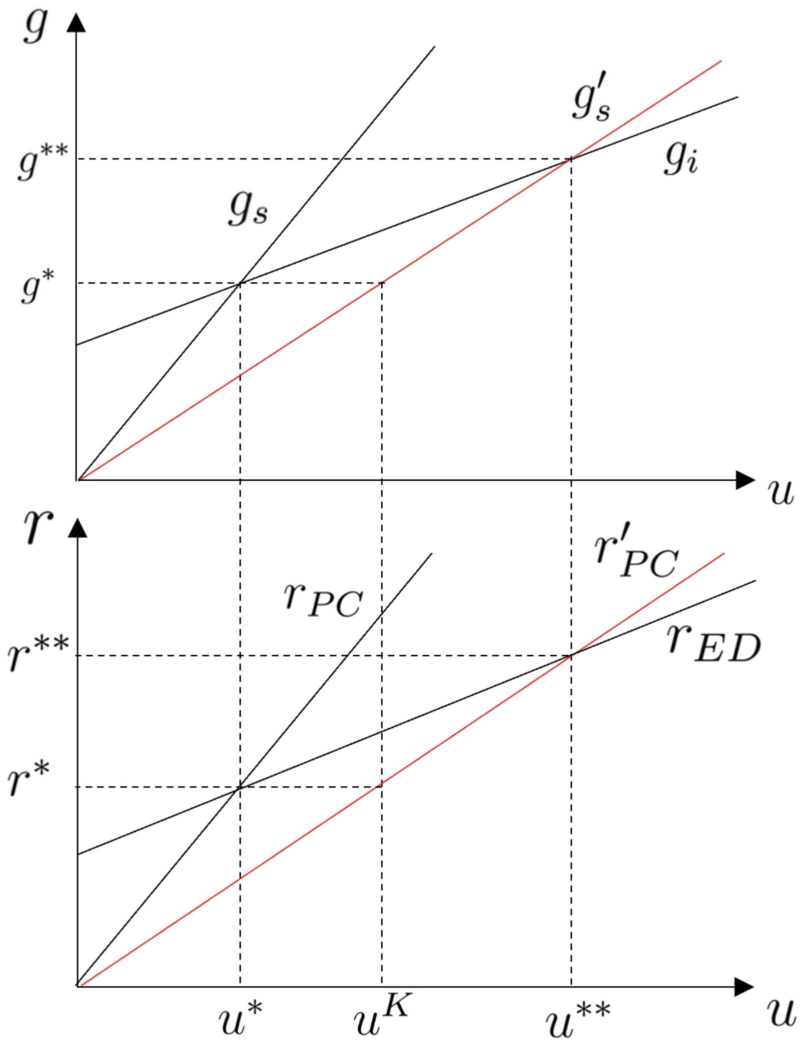

Figure 4.3: The Neo-Kaleckian Growth Model.

The distributional shift implies higher sales and a higher demand for consumption goods, which, at the same rate of accumulation g^*. The new *short-run semi-equilibrium* point is related to a new capacity utilization $u^K = \frac{\alpha+\beta(u^e-u_n)}{(s_r\pi/v)}$ (where $u \neq u^e$, with the superscript K of *keynesian equilibrium*), which assumes that firms are always able to adjust production to sales within the same period ($S = I$). However, firms investment decisions are made on the basis of the expected capacity utilization u^e, as in equation (4.29). Hence the adjustment mechanism of the expected utility capacity utilization is

$$\dot{u^e} = \mu_1(u^K - u^e), \text{ for } \mu_1 > 0 \tag{4.36}$$

At point u^* we have that $u^* = u^e \Rightarrow \dot{u^e} = 0$, on the other hand, at u^K we have that

$u^K > u^e \Rightarrow \dot{u^e} > 0$. Higher sales lead firms to anticipate higher rates of utilization, which will induce them to raise their rate of accumulation until the economy reaches a new equilibrium g^{**} where $u^{**} = u^e = u^K$. This new equilibrium lustrates the *paradox of thrift* (see Section 3.2) in terms of the Neo-Kaleckian model: the reduction in the profit share (π) implicated a reduction in the the aggregate propensity to save ($s = s_r\pi$), as defined in equation (4.22), which in turn lead to an increase in the long-period rates of accumulation (g^{**}) an capacity utilization (u^{**}).

If, by a fluke $u^* = u^n$, then, wouldn't be find that $u^{**} > u_n$ and $g_a = g^{**} > g_w$? Yes, in fact we would, and under the mechanics of the Harrod-Domar model (see Section 4.1) we would fin ourselves again under the divergence path of *Harrodian instability*: this is only prevented under the heroic assumption that $u^{**} = u_n$ becomes the *new normal*. However, we can identify a second paradoxical result emanating from this model: the *paradox of costs*. The reduction in the profit share (and hence the increased in the wage share ω) leads to an increase in the real wage, and therefore also into higher costs of production and a higher long-period profit rate.

4.5 Sraffian Supermultiplier

This section follows (Cesaratto et al., 2003) and Freitas and Serrano (2015). The model is derived from the following set of equations, assuming that each variable is in itself a function time:

$$Y = C + I + Z - M \tag{4.37}$$

$$C = c_1(1-\tau)Y \tag{4.38}$$

$$I = hY = v(\delta + g_e)Y \tag{4.39}$$

$$M = mY \tag{4.40}$$

$$Z = c_0 + G + B + X \tag{4.41}$$

$$s = 1 - c_1(1-\tau) + m = \frac{\partial S}{\partial Y} \tag{4.42}$$

$$Y = \left(\frac{1}{s-h}\right) Z = \left(\frac{1}{s - v(\delta + g_e)}\right) Z \tag{4.43}$$

Where Y is the output of the economy defined by (i) consumption (C), which depends on the marginal propensity to consume (c_1) and disposable income $(1-\tau)Y$; (ii) investment, which is a function of the marginal propensity to invest h and output at the same period;[2] (iii) the autonomous components of demand Z, which are the sum of autonomous consumption c_0, government spending G, autonomous business expenditure B and exports X; (iv) and imports, which are a function of the marginal propensity to import m and current output.

Remembering that $S = I$, equation (4.42) describes the marginal propensity to consume. Let $\frac{S}{y} = s_a$, that is, the average propensity to save. Notice that

$$S = I = Y - C - Z + M = Y[1 - c_1(1-\tau) + m] - Z$$
$$\Rightarrow s_a = s - \frac{Z}{Y} = s(\frac{s_a}{s}) = s(\frac{I}{Y} \cdot \frac{1}{s}) = h$$

In the model the existence of the autonomous components of demand imply, by construction an inequality between the marginal propensity to consume s and the average propensity to save, which, in turn, is equal to the average propensity to invest h. Solving the system yields equation (4.43), which is the *demand-determined* level of output in a long-period position, where the term within the parenthesis is the *supermultiplier* (exogenous and constant, so that it only has level effects on the trajectory of Y) that captures the effects on the level of output associated with both *induced* consumption (s) and investment (h). We can start to introduce growth dynamics:

$$Y_K = \left(\frac{1}{v}\right) K \tag{4.44}$$

$$u = \frac{Y}{Y_k} \tag{4.45}$$

$$g_{Y_K} = g_K = \left(\frac{h}{v}\right) u - \delta \tag{4.46}$$

$$\dot{u} = u(g_Y - g_{Y_K}) \tag{4.47}$$

[2] Equation (4.39) also contains an alternative definition of investment in which the function is such that amortization and expansion depend on current effective demand and long-term expectation. Where v is the usual desired capital-output ratio, δ is depreciation rate (or *replacement coefficient*) and g_e is the expected average rate of growth of normal effective demand over the life of the investment that is currently being installed (Cesaratto et al., 2003, p. 42).

Where equation (4.44) defines the level of the capacity output of the economy (Y_K), which depends on the level of the capital stock and on the technical/desired capital-output ratio v (which is constant over time). Hence, the rate of growth of capacity output (g_{Y_K}) is equal to the rate of growth of capital accumulation (g_k), as seen in equation (4.46): which depends on the average propensity to invest (h), the capital-output ratio (v) the actual degree of capacity utilization (u) and the depreciation rate of capital (δ).[3] Finally, equation (4.47) is obtained differentiating $\ln(u) = \ln(Y) - \ln(Y_K)$ with respect to time. Let g_Z be the rate of growth of the autonomous components of demand. Since the marginal propensity to save (s) is constructed only using exogenous parameters, and since h is also considered given, it is easy to see that:

$$\dot{Y} = \left(\frac{1}{s-h}\right)\dot{Z}$$

$$g_Y = \frac{\left(\frac{1}{s-h}\right)\dot{Z}}{\left(\frac{1}{s-h}\right)Z} = \frac{\dot{Z}}{Z} = g_Z$$

$$g_Z = g_Y = g_C = g_I \tag{4.48}$$

More over, we can also see that $g_K = g_I = g_Z$, from $\dot{K} = I - \delta K$ we can see that:

$$g_K = \frac{I}{K} - \delta$$

$$\dot{g_K} = \frac{\dot{I}K - \dot{K}I}{K^2} = \frac{\dot{I}}{K} - \frac{I}{K}g_K = \frac{g_I}{K/I} - \frac{I}{K}g_K$$

$$= \frac{I}{K}g_I - \frac{I}{K}g_K = \frac{I}{K}(g_I - g_K) \text{ and since } \frac{I}{K} = g_K - \delta$$

$$\Rightarrow \dot{g_K} = (g_K - \delta)(g_I - g_K) \tag{4.49}$$

By equation (4.49), we can observe that if $g_I > g_K \Rightarrow \dot{g_K} > 0$ and if $g_K > g_I \Rightarrow \dot{g_K}$. Hence, g_K only remains constant over time when $g_I = g_K$, which in turn also implies that $g_K = g_Z$. The long period equilibrium capacity utilization will then be given, manipulating equation (4.46), by:

$$u^* = \frac{v(g_Z + \delta)}{h} \tag{4.50}$$

[3] This is derived from $\dot{K} = I - \delta K$, which can be transformed via $\frac{\dot{K}}{K} = g_K = \frac{I}{K} - \delta \Rightarrow g_K = (\frac{I}{K})(\frac{Y}{Y})(\frac{Y_K}{Y_K}) - \delta$, which finally yields $g_K = (\frac{I}{Y})(\frac{Y_K}{K})(\frac{Y}{Y_K}) - \delta = \left(\frac{h}{v}\right)u - \delta$

This implies that changes in the average propensity to invest (h) will permanently alter, inversely, the long-period capacity utilization u^*. And, also, that changes in the rate of growth of the autonomous components of demand will also imply permanent changes in the level of equilibrium capacity utilization. By equation (4.47), it is evident that if $g_Y \neq g_K \Rightarrow g_K \rightarrow g_Y$, such that when $g_K = g_I = g_Z$ then $\dot{u} = 0$.

The long-period position, then, does not imply a *fully adjusted* position where $u = u^* = u_n$, where u_n is the normal capacity utilization of the economy. For Cesaratto et al. (2003, p. 44) the actual degree of capacity utilization (u) tends to move to its normal level (u_n) as the distance between actual and expected growth rates of effective demand narrows (under the assumption that firms revise and adjust g_e), and the size and growth rate of capacity output Y_K adjusts to the trend of effective demand: hence the expected and actual rates of growth of the economy tend to converge to the rate of growth of autonomous demand: $g_e = g_z$. The fully adjusted position can be defined as:

$$Y^* = \left(\frac{1}{s - v(\delta + g_Z)}\right) Z \tag{4.51}$$

This last equation describes a *center of gravitation* of the accumulation process, whose pace is determined by the trend that the autonomous components of demand set. In this model the ultimate causes of growth are not found in the model, but rather, in the institutional, political and social determinants of the components of autonomous demand.

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